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AN EXPONENTIAL REPRESENTATION FOR SOME INTEGRALS WITH RESPECT TO LEBESGUE-POISSON MEASURE

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Abstract. We prove a theorem that allows to simplify some combinatorial calculations. An example of application of this theorem in statistical mechanics is given.

1. Introduction

This short article is devoted to one mathematical aspect of infinite dimensional analysis which is used in solving problems of statistical mechanics. This aspect is a use of mathematical properties of integrals with respect to a Poisson measure (more exactly with respect to a so-called Lebesgue-Poisson measure $\lambda_{z\sigma}$ which we define later). A Poisson measure is defined on the space of locally finite configurations of the Euclidean space $\mathbb{R}^d$ (see, e.g., [3, 9, 10, 15, 11, 12, 29]) and used in the construction of a Gibbs measure for infinite systems of point particles. Different aspects of analysis on the configuration space as well as the problems of construction of different measures were developed in numerous publications (see, e.g., [17, 18, 1, 2, 16, 13, 4, 5], see also the latest review [14]) and references therein.

The Poisson measure belongs to the class of measures that have the property of infinite divisibility ([6], Chapter 4.4, see also [19]). A similar property can be written also for the integrals with respect to Lebesgue-Poisson measures (see, e.g., [26], Eq. (2.15)). It has been an extremely useful technical tool for constructing new types of cluster expansions [23, 7, 8, 24], the use of which has made it possible to simplify the proof of superstable estimates for correlation functions (see [25, 21, 22]).

In this paper, we want to show another advantage of application of integrals with respect to the Lebesgue-Poisson measure, which greatly simplified cumbersome combinatorial formulas for summation with constraints. Of course, formula (2.10), which we prove in Section 3. is not new. In statistical mechanics, the proof goes back to the algebraic technique used by Ruelle [27], Chapter 4.4 (see also [30, 31] and details in [28] or in the latest article [5]). It also follows from [16] (Corollary 2.1.4), but with additional combinatorial resummation. In this article we propose another very short proof of formula (2.10) using the well known integral identity (3.11). An application to representation of grand partition function in statistical mechanics is given.

2. Definitions and main result

Let $\mathbb{R}^d$ be a d-dimensional Euclidean space. By $\mathcal{B}(\mathbb{R}^d)$ we denote the family of all Borel sets and by $\mathcal{B}_c(\mathbb{R}^d)$ denote the systems of all sets in $\mathcal{B}(\mathbb{R}^d)$ which are bounded.

The set of locally finite subsets in $\mathbb{R}^d$ we call the configuration space,

$$\Gamma = \Gamma_{\mathbb{R}^d} := \left\{ \gamma \subset \mathbb{R}^d \middle| \, |\gamma \cap \Lambda| < \infty, \text{ for all } \Lambda \in \mathcal{B}_c(\mathbb{R}^d) \right\}, \tag{2.1}$$

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where $|A|$ denotes the cardinality of the set A. For any $\Lambda \in \mathcal{B}_c(\mathbb{R}^d)$ we denote by γ_Λ the intersection of γ and Λ. We also need to define the space of finite configurations Γ_0 in $\mathbb{R}^d$,

$$\Gamma_0 = \bigsqcup_{n\in\mathbb{N}_0} \Gamma^{(n)}, \quad \Gamma^{(n)} := \{\eta \in \Gamma \mid |\eta| = n\}, \quad \mathbb{N}_0 = \mathbb{N} \cup \{0\}, \tag{2.2}$$

and the space of finite configurations in Λ,

$$\Gamma_\Lambda := \{\, \gamma \in \Gamma_0 |\ \gamma \subset \Lambda\}\,. \tag{2.3}$$

The corresponding σ-algebras of these spaces will be denoted by $\mathcal{B}(\Gamma_0)$ and $\mathcal{B}(\Gamma_\Lambda)$ (see, e.g., details in [1]).

Let σ be the Lebesgue measure on $\mathcal{B}(\mathbb{R}^d)$ and for any $n \in \mathbb{N}$ the product measure $\sigma^{\otimes n}$ can be considered as a measure on

$$\widetilde{(\mathbb{R}^d)^n} = \left\{ (x_1, \ldots, x_n) \in (\mathbb{R}^d)^n \middle|\ x_k \neq x_l \text{ if } k \neq l \right\}$$

and hence as a measure $\sigma^{(n)}$ on $\Gamma^{(n)}$ through the map

$$\mathrm{sym}_n : \widetilde{(\mathbb{R}^d)^n} \ni (x_1, \ldots, x_n) \mapsto \{x_1, \ldots, x_n\} \in \Gamma^{(n)}.$$

For any $z > 0$ define the Lebesgue-Poisson measure $\lambda_{z\sigma}$ on $\mathcal{B}(\Gamma_0)$ by the formula

$$\lambda_{z\sigma} := \sum_{n\geq 0} \frac{z^n}{n!} \sigma^{(n)}. \tag{2.4}$$

The restriction of $\lambda_{z\sigma}$ to $\mathcal{B}(\Gamma_\Lambda)$ we also denote by $\lambda_{z\sigma}$. For any $\mathcal{B}(\Gamma_\Lambda)$-measurable bounded function F an integral with respect to the measure $\lambda_{z\sigma}$ can be def ind by the formula

$$\int_{\Gamma_\Lambda} F(\gamma)\lambda_{z\sigma}(d\gamma) = \sum_{n=0}^{\infty} \frac{z^n}{n!} \int_\Lambda \cdots \int_\Lambda F(\{x_1, \ldots, x_n\})\sigma(dx_1)\cdots\sigma(dx_n). \tag{2.5}$$

Any function $F : \Gamma_\Lambda \mapsto \mathbb{R}$ or $F : \Gamma_0 \mapsto \mathbb{R}$ can be defined by an infinite sequence of symmetric functions $\{F_n\}_{n\geq 0}$, so that for any $\gamma = \{x_1, \ldots, x_n\} \in (\mathbb{R}^d)^{\otimes n}$ we have

$$F(\gamma) = F(\{x_1, \ldots, x_n\}) = F_n(x_1, \ldots, x_n) = F(x_{\pi(1)}, \ldots, x_{\pi(n)}), \quad F(\emptyset) = F_0 \in \mathbb{R}, \tag{2.6}$$

where $\pi \in \mathcal{P}_n$ and $\mathcal{P}_n$ is the group of permutations of n elements.

We consider functions on the configuration space Γ_Λ, which have the form

$$\Phi(\gamma) \ = \sum_{k=1}^{|\gamma|} \ \sum_{\{\gamma_1,\ldots,\gamma_k\}\subset\gamma}^{*} F(\gamma_1)F(\gamma_2)\cdots F(\gamma_k), \quad \Phi(\emptyset) = 1, \tag{2.7}$$

where the asterisk over the sum means that the sum is taken over all partitions of the set γ into k non-empty disjoint subsets, i.e.,

$$\bigcup_{j=1}^{k} \gamma_j = \gamma, \quad \gamma_i \bigcap \gamma_j \ = \ \emptyset \quad \text{for all} \quad i \neq j, \quad \gamma_i \neq \emptyset, \quad i, j \in \{1, \ldots, k\}. \tag{2.8}$$

The main result of the work is the following theorem.

Theorem 2.1. *Let the function Φ have the form as in* (2.7) *and the functions F_n (see* (2.6)*) satisfy the following estimates:*

$$\int_\Lambda \cdots \int_\Lambda F_n(x_1, \ldots, x_n)\, dx_1 \cdots dx_n \ \leq \ c^n C_\Lambda n!, \tag{2.9}$$

where the constant c does not depend on Λ. Then

$$\int_{\Gamma_\Lambda} \Phi(\gamma)\lambda_{z\sigma}(d\gamma) \ = \ e^{\int_{\Gamma_\Lambda\setminus\{\emptyset\}} F(\gamma)\lambda_{z\sigma}(d\gamma)}, \tag{2.10}$$

for $0 < z < 1/2c$.

3. Proof of Theorem 2.1

Let us introduce the following function:

$$\Phi(\alpha;\gamma) \;=\; \sum_{k=1}^{|\gamma|} \frac{\alpha^k}{k!}\Phi_k(\gamma), \quad \alpha\in\mathbb{R}, \tag{3.1}$$

where

$$\Phi_k(\gamma) \;=\; \sum^{*}_{(\gamma_1,\dots,\gamma_k)\subset\gamma} F_n(\gamma_1)F_n(\gamma_2)\cdots F_n(\gamma_k) \tag{3.2}$$

and the functions F_n are as in (2.7)–(2.10). Unlike formula (2.7) in the formula (3.2) the summation is taken over all ordered collections of partitions which satisfy (2.8). It is clear that $\Phi(\gamma) \;=\; \Phi(1;\gamma)$. We start the proof with the following lemma.

Lemma 3.1. *Let the function F in the definition* (2.7) *satisfy the assumption* (2.9). *Then, for any $\alpha\in\mathbb{R}$ the function $\Phi(\alpha;\cdot)$ is integrable on Γ_Λ with respect to the measure $\lambda_{z\sigma}$ and, moreover, for any $0<z<1/2c$ the function I given by the formula*

$$I(\alpha) \;=\; \int_{\Gamma_\Lambda}\Phi(\alpha;\gamma)\lambda_{z\sigma}(d\gamma), \quad \alpha\in\mathbb{R} \tag{3.3}$$

is analytic and can be represented by a Maclaurin series for any $\alpha\in(-R,R)$, and $R>0$.

Proof. It is easy to calculate that

$$\begin{aligned} I^{(m)}(\alpha) &= \frac{d^m I(\alpha)}{d\alpha^m} = \int_{\Gamma_\Lambda} 1\!\!1_{A_m}(\gamma)\sum_{k=m}^{|\gamma|}\frac{\alpha^{k-m}}{(k-m)!}\Phi_k(\gamma)\lambda_{z\sigma}(d\gamma) \\ &= \sum_{N=m}^{\infty}\frac{z^N}{N!}\sum_{k=m}^{N}\frac{\alpha^{k-m}}{(k-m)!}\int_{\Lambda^N}(dx)^N \sum^{*}_{(\gamma_1,\dots,\gamma_k)\subset\{x_1,\dots,x_N\}} F(\gamma_1)F(\gamma_2)\dots F(\gamma_k), \end{aligned} \tag{3.4}$$

where $1\!\!1_{A_m}(\gamma)$ is an indicator of the set

$$A_m \;:=\; \{\gamma\in\Gamma_\Lambda \mid |\gamma|\geq m\}. \tag{3.5}$$

Denote $|\gamma_j| \;=\; n_j,\;\; j \;=\; \overline{1,k}$. Then the sum with asterisk in (3.4) is a sum over all possible partitions of the configuration $\{x_1,\dots,x_N\}$ into k disjoint configurations with a fixed number of elements. $n_1,\dots,n_k$, and a sum over all possible values $n_j\geq 1$ with the constraint $n_1+\cdots+n_k \;=\; N$. The sum over all possible partitions is a classic combinatorial problem and it has $N!/n_1!\cdots n_k!$ elements, and

$$\sum_{\substack{\{n_1,\dots,n_k\geq 1\}\\ n_1+\cdots+n_k=N}} 1 \;\leq\; \sum_{\substack{\{n_1,\dots,n_k\geq 0\}\\ n_1+\cdots+n_k=N}} 1 \;=\; C^N_{N+k-1} \;<2^{N+k}.$$

Then, taking into account (2.9) we obtain

$$|I^{(m)}(\alpha)| \;\leq\; \sum_{N=m}^{\infty}(2cz)^N\sum_{k=m}^{N}\frac{|\alpha|^{k-m}(2C_\Lambda)^k}{(k-m)!} \;<\; \frac{(4czC_\Lambda R)^m}{(1-2cz)R^m}e^{2|\alpha|C_\Lambda} \;<\; M\frac{m!}{R^m} \tag{3.6}$$

with

$$M \;=\; M(R) \;=\; \frac{e^{2(1+2cz)RC_\Lambda}}{1-2cz}, \tag{3.7}$$

and any $R>0$. □

Therefore, we can write for the function $I(\alpha)$

$$I(\alpha) \;=\; \sum_{m=0}^{\infty}\frac{\alpha^m}{m!}I^{(m)}(0), \quad |\alpha|<R. \tag{3.8}$$

Proof of the Theorem 2.1.

It is clear from (3.3) that $I(1)$ is the left-hand side of (2.10). So, to prove the theorem we need to show that right-hand side of (3.8) is the exponent (2.10) at $\alpha = 1$. From the equation (3.4),

$$I^{(m)}(0) \;=\; \int_{\Gamma_\Lambda} 1\!\!1_{A_m}(\gamma)\Phi_m(\gamma)\lambda_{z\sigma}(d\gamma). \tag{3.9}$$

Wright the function $\Phi_m(\gamma)$ (see (3.2)) in the form

$$\Phi_m(\gamma) \;=\; \sum_{\eta\subseteq\gamma} 1\!\!1_{A_1}(\eta)F(\eta)1\!\!1_{A_{m-1}}(\gamma\setminus\eta)\Phi_{m-1}(\gamma\setminus\eta). \tag{3.10}$$

Note that in (3.10) the sum starts from the empty configuration $\eta = \emptyset$, but the corresponding term is equal to zero as $1\!\!1_{A_1}(\emptyset) = 0$ by definition. Inserting (3.10) into the right-hand side of (3.9) we use the formula (see, e.g., [16], Lemma 2.1.3)

$$\int_{\Gamma_\Lambda} G(\gamma)\sum_{\eta\subseteq\gamma} H(\eta,\gamma\setminus\eta)\lambda_{z\sigma}(d\gamma) \;=\; \int_{\Gamma_\Lambda}\int_{\Gamma_\Lambda} G(\eta\cup\gamma)H(\eta,\gamma)\lambda_{z\sigma}(d\eta)\lambda_{z\sigma}(d\gamma), \tag{3.11}$$

which is true for any $G, H \in L^1(\Gamma_\Lambda, \lambda_{z\sigma})$. It is easy to obtain this formula using the definition of the measure $\lambda_{z\sigma}$ (see (2.5)). Then taking

$$G(\gamma) \;=\; 1\!\!1_{A_m}(\gamma), \quad \text{and} \quad H(\eta,\gamma\setminus\eta) \;=\; 1\!\!1_{A_1}(\eta)F(\eta)1\!\!1_{A_{m-1}}(\gamma\setminus\eta)\Phi_{m-1}(\gamma\setminus\eta), \tag{3.12}$$

and taking into account that $1\!\!1_{A_m}(\gamma\cup\eta)1\!\!1_{A_1}(\eta)1\!\!1_{A_{m-1}}(\gamma) = 1\!\!1_{A_1}(\eta)1\!\!1_{A_{m-1}}(\gamma)$ we obtain

$$\begin{aligned} I^{(m)}(0) &= \int_{\Gamma_\Lambda}\int_{\Gamma_\Lambda} 1\!\!1_{A_m}(\gamma\cup\eta)1\!\!1_{A_1}(\eta)F(\eta)1\!\!1_{A_{m-1}}(\gamma)\Phi_{m-1}(\gamma)\lambda_{z\sigma}(d\eta)\lambda_{z\sigma}(d\gamma) \\ &= \int_{\Gamma_\Lambda} 1\!\!1_{A_1}(\eta)F(\eta)\lambda_{z\sigma}(d\eta)\int_{\Gamma_\Lambda} 1\!\!1_{A_{m-1}}(\gamma)\Phi_{m-1}(\gamma)\lambda_{z\sigma}(d\gamma). \end{aligned} \tag{3.13}$$

Therefore

$$I^{(m)}(0) \;=\; \left(\int_{\Gamma_\Lambda} 1\!\!1_{A_1}(\eta)F(\eta)\lambda_{z\sigma}(d\eta)\right) I^{(m-1)}(0). \tag{3.14}$$

Iterating this equation we obtain with (3.8) and for $\alpha = 1$ the proof of the Theorem 2.1.

4. Application to statistical mechanics

We will use Theorem 2.1 to obtain so-called Mayer expansions for pressure and density of an infinite system of point particles interacting via a pair potential ϕ which is a continuous function on $\mathbb{R}_+ \setminus \{0\}$ and which satisfies the following conditions of stability and regularity.

(A1) Stability: *the potential ϕ is called stable, if the energy of any configuration $\gamma \in \Gamma_\Lambda$ satisfies the following inequality:*

$$U(\gamma) \;=\; \sum_{\{x,y\}\subset\gamma} \phi(|x-y|) \;\ge\; -B|\gamma|, \quad |\gamma| \ge 2 \tag{4.1}$$

with some constant $B \ge 0$.

(A2) Regularity:

$$C(\beta) \;:=\; \int_{\mathbb{R}^d} |e^{-\beta\phi(|x|)} \;-\; 1|dx \;<\; \infty. \tag{4.2}$$

Pressure in the system is a function of activity z and inverse temperature $\beta = 1/kT$, where k is Boltzmann's constant and is given by the following formula (see, e.g., [28], Theorem 4.3.1):

$$p(z,\beta) \;=\; \lim_{\Lambda\uparrow\mathbb{R}^d} p^\Lambda(z,\beta) \;:=\; \lim_{\Lambda\uparrow\mathbb{R}^d} \frac{1}{\beta\sigma(\Lambda)} \log Z_\Lambda(z,\beta), \tag{4.3}$$

where $Z_\Lambda(z,\beta)$ is the great partition function. It can be written as the Lebesgue-Poisson measure integral of the Boltzmann functional (see, e.g., [28], Chapter 4),

$$Z_\Lambda(z,\beta) := \int_{\Gamma_\Lambda} e^{-\beta U(\gamma)} \lambda_{z\sigma}(d\gamma), \tag{4.4}$$

where

$$U(\gamma) = \begin{cases} 0, & \text{for } |\gamma| = 0 \vee 1, \\ \sum_{\{x,y\}\subset\gamma} \phi(|x-y|) := \sum_{\{x,y\}\subset\gamma} \phi_{xy}, & \text{for } |\gamma| \geq 2. \end{cases} \tag{4.5}$$

In order to apply Theorem 2.1 we can write the functional $e^{-\beta U(\gamma)}$ in the form

$$e^{-\beta U(\gamma)} = \begin{cases} 1, & \text{for } |\gamma| = 0 \vee 1, \\ \prod_{\{x,y\}\subset\gamma}(C_{xy} + 1), & \text{for } |\gamma| \geq 2, \end{cases} \tag{4.6}$$

where $C_{xy} := e^{-\beta\phi_{xy}} - 1$.

The product in (4.6) can be conveniently written as a sum of contributions from the graph whose vertices are points of the configuration γ. The contribution from the vertex is one, and the contribution of the line $l = l_{xy}$ connecting the two points $x, y \in \gamma$ is the function C_{xy}. So, the contribution of any graph is a product of the functions $C_{l_{xy}} = C_{xy}$ over all internal lines $\mathcal{L}(G)$ of the graph G. It is clear that for any configuration $\gamma \in \Gamma_\Lambda$ the product in (4.6) includes $2^{N(N-1)/2}$ graphs, where $N = |\gamma|$. If we denote any graph by $G = G(\gamma)$, and the whole set of such graphs by $\mathcal{G}(\gamma)$, the Boltzmann function can be represented as the following sum:

$$e^{-\beta U(\gamma)} = \sum_{G\in\mathcal{G}(\gamma)} \prod_{\{x,y\}\in\mathcal{L}(G)} C_{xy}. \tag{4.7}$$

Each graph G can be represented as the composition of k graphs $G_i^T (i \in \{1,\ldots,k\})$ which are connected graphs,

$$G = G_1^T * \cdots * G_k^T. \tag{4.8}$$

The number k is called the order of disconnectedness of the graph G $(1 \ \leq k \ N = |\gamma|)$. Graphs with the order of disconnectedness $k = 1$ are connected graphs. The contribution of any graph G with the order of disconnectedness $k > 1$ is the product of k contributions from every graph G_i^T, $i = \overline{1,k}$. With every such graph $G \in \mathcal{G}$ one can connect a partition configuration γ on subconfigurations $\{\gamma_1,\ldots,\gamma_k\}$, which satisfy the conditions (2.8), but for every such partition there are many graphs G with the same order k of disconnectedness.

One can perform in the sum (4.7) the following resummation. We split the sum (4.7) into $N = |\gamma|$ groups with a fixed order of disconnectedness. Then, set of graphs with fixed k is divided into groups, the set of graphs in which is based on fixed vertices that correspond to the partition configuration γ on k subconfigurations $\{\gamma_1,\ldots,\gamma_k\}$. The last step in this resummation is the following. First we sum all graphs that have the same contributions, corresponding to partitions $\{\gamma_2,\ldots,\gamma_k\}$ and different contributions corresponding to γ_1. As a result, we obtain a sum of contributions, each term of which will have the same first factor

$$\Phi^T(\gamma_1) = \begin{cases} 1, & \text{for } |\gamma_1| = 1, \\ \sum_{G^T\in\mathcal{G}^T(\gamma_1)} \prod_{\{x,y\}\in\mathcal{L}(G^T)} C_{xy}, & \text{for } |\gamma_1| \geq 2, \end{cases} \tag{4.9}$$

where $\mathcal{G}^T(\gamma_1)$ is the set of all connected graphs with vertices in the points of the configuration γ_1. The functions $\Phi^T(\gamma)$ are called Ursell functions (see, e.g., [28], Chapter 4.4.2) which is connected with truncated correlation functions (see, e.g., [20]. Then we sum all graphs that have the same contributions corresponding to the partitions $\{\gamma_3,\ldots,\gamma_k\}$ and different contributions corresponding to γ_2 and so on. As a result, we finally get

$$e^{-\beta U(\gamma)} = \sum_{k=1}^{|\gamma|} \sum_{\{\gamma_1,\ldots,\gamma_k\}\subset\gamma}^{*} \Phi^T(\gamma_1)\cdots\Phi^T(\gamma_k), \tag{4.10}$$

where the sum with asterisk has the same meaning as in (2.7), and for the great partition function we have the following representation:

$$Z_\Lambda(z,\beta) := \int_{\Gamma_\Lambda} \sum_{k=1}^{|\gamma|} \sum_{\{\gamma_1,\dots,\gamma_k\}\subset\gamma}^{*} \Phi^T(\gamma_1)\cdots\Phi^T(\gamma_k)\lambda_{z\sigma}(d\gamma). \tag{4.11}$$

To apply Theorem 2.1 with $F(\gamma) = \Phi^T(\gamma)$ we need an estimate for $\Phi^T(\gamma)$. One can find bounds on these functions in [28], Chapter 4, Sec. 4.4.6, which is

$$\int_\Lambda \cdots \int_\Lambda |\Phi^T(\{x_1,\dots,x_n\})| dx_2\cdots dx_n \ \le\ (n-1)! e^{-2\beta B} \left(e^{2\beta B+1} C(\beta)\right)^{n-1}. \tag{4.12}$$

Now we can apply the theorem 2.1 with $C_\Lambda = \sigma(\Lambda) C(\beta)^{-1} e^{-4\beta B-1}$ and $c = e^{2\beta B+1} C(\beta)$ to obtain the following representation for great partition function:

$$Z_\Lambda(z,\beta) \ =\ e^{\int_{\Gamma_\Lambda\setminus\{\emptyset\}} \Phi^T(\gamma)\lambda_{z\sigma}(d\gamma)}. \tag{4.13}$$

This formula and estimate (4.12) prove the existence and analyticity of the pressure. Indeed, using definition (4.3) we have

$$\begin{aligned} p^\Lambda(z,\beta) \ &=\ \frac{1}{\beta\sigma(\Lambda)} \log Z_\Lambda(z,\beta) = \frac{1}{\beta\sigma(\Lambda)} \int_{\Gamma_\Lambda\setminus\{\emptyset\}} \Phi^T(\gamma)\lambda_{z\sigma}(d\gamma) \\ &= \frac{1}{\beta\sigma(\Lambda)} \sum_{n=1}^{\infty} \frac{z^n}{n!} \int_\Lambda \cdots \int_\Lambda |\Phi^T(\{x_1,\dots,x_n\})|\, dx_1\cdots dx_n. \end{aligned}$$

As a result the existence and analyticity of the pressure follows from the existence of integrals of the function $\Phi^T(\{x_1,\dots,x_n\})$ with respect to the variables $x_2,\dots,x_n$ (see (4.12)). A similar result holds also for the density of the infinite system. This follows from the expression for the density (see [28], Chapter 4, Sec. 4.4.7)

$$\rho \ =\ z + \sum_{n=2}^{\infty} \frac{z^n}{(n-1)!} \int_{\mathbb{R}^d} \cdots \int_{\mathbb{R}^d} |\Phi^T(\{x_1,\dots,x_n\})|\, dx_2\cdots dx_n \tag{4.14}$$

and estimate (4.12).

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References

1. S. Albeverio, Yu. G. Kondratiev, and M. Röckner, *Analysis and geometry on configuration spaces*, J. Funct. Anal. **154** (1998), 444–500.
2. S. Albeverio, Yu. G. Kondratiev, and M. Röckner, *Analysis and geometry on configuration spaces: The Gibbsian case*, J. Funct. Anal. **157** (1998), 242–291.
3. Yu. M. Berezansky, *Poisson infinite-dimensional analysis as an example of analysis related to generalized translation operators*, Funktsional. Anal. i Prilozhen. **32** (1998), no. 3, 65–70. (Russian); English transl. Funct. Anal. Appl. **32** (1998), no. 3, 195–198.
4. D. L. Finkelshtein, *On convolutions on configuration spaces.* I. *Spaces of finite configurations*, Ukrain. Mat. Zh. **64** (2012), no. 11, 1547–1567. (Ukrainian); English transl. Ukrainian Math. J. **64** (2013), no. 11, 1752–1775.
5. D. L. Finkelshtein, *On convolutions on configuration spaces.* II. *Spaces of locally finite configurations*, Ukrain. Mat. Zh. **64** (2012), no. 12, 1699–1719. (Ukrainian); English transl. Ukrainian Math. J. **64** (2013), no. 12, 1919–1944.
6. I. M. Gelfand, N. Ya. Vilenkin, *Generalized Functions.* Vol. 4*: Applications of Harmonic Analysis*, Academic Press, New York, 1964.
7. R. Gielerak, A. L. Rebenko, *Poisson field representation in the statistical mechanics of continuous systems*, Oper. Theory Adv. Appl. **70** (1994), 219–226.
8. R. Gielerak, A. L. Rebenko, *On the Poisson integrals representation in the classical statistical mechanics of continuous systems*, J. Math. Phys. **37** (1996), 3354–3374.
9. Y. Ito and I. Kubo, *Calculus on Gaussian and Poisson white noise*, Nagoya Math. J. **111** (1988), 41–84.

10. N. A. Kachanovsky, *On biorthogonal approach to a construction of non-Gaussian analysis and application to the Poisson analysis on the configuration space*, Methods Funct. Anal. Topology **6** (2000), no. 2, 13–21.
11. Yu. G. Kondratiev, T. Kuna, and M. J. Oliveira, *Analytic aspects of Poissonian white noise analysis*, Methods Funct. Anal. Topology **8** (2002), no. 4, 15–48.
12. Yu. G. Kondratiev, T. Kuna, and M. J. Oliveira, *On the relations between Poissonian white noise analysis and harmonic analysis on configuration spaces*, J. Funct. Anal. **213** (2004), no. 1, 1–30.
13. Yu. G. Kondratiev and O. Kutoviy, *On the metrical properties of the configuration space*, Math. Nachr. **279** (2006), 774–783.
14. Yu. G. Kondratiev, T. Pasurek, and M. Röckner, *Gibbs measures of continuous systems: an analytic approach*, Rev. Math. Phys. **24** (2012), no. 10, 1250026 (54 pages).
15. Yu. G. Kondratiev, J. L. Silva, L. Streit, and G. Us, *Analysis on Poisson and gamma spaces*, Infin. Dimens. Anal. Quantum Probab. Relat. Top. **1** (1998), no. 1, 91–117.
16. T. Kuna, *Studies in configuration space analysis and applications*, Bonner Mathematische Schriften [Bonn Mathematical Publications], 324. Universität Bonn Mathematisches Institut, Bonn, 1999. Dissertation, Rheinische Fridrich-Universität Bonn, Bonn, 1999.
17. A. Lenard, *States of classical statistical mechanical systems of infinitely many particles.* I, Arch. Rational Mech. Anal. **59** (1975), 219–239.
18. A. Lenard, *States of classical statistical mechanical systems of infinitely many particles.* II, Arch. Rational Mech. Anal. **59** (1975), 241–256.
19. K. Matthes, J. Kerstan, and J. Mecke, *Infinitely divisible point processes*, John Wiley and Sons, Chichester—New York—Brisbane, 1978.
20. R. A. Minlos, S. K. Pogosjan, *Estimates of Ursell functions, group functions, and their derivatives*, Teoret. Mat. Fiz. **31** (1977), no. 2, 199–213. (Russian); English transl. Theoret. Math. Phys. **31** (1977), no. 2, 408–418.
21. S. N. Petrenko, A. L. Rebenko, *Superstable criterion and superstable bounds for infinite range interaction I: two-body potentials*, Methods Funct. Anal. Topology **13** (2007), 50–61.
22. S. N. Petrenko, A. L. Rebenko, *Superstable criterion and superstable bounds for infinite range interaction II: many-body potentials*, Zb. prac' Inst. mat. NAN Ukr. **6** (2009), no. 1, 191–208.
23. A. L. Rebenko, *Poisson measure representation and cluster expantion in classical statistical mechanics*, Comm. Math. Phys. **151** (1993), 427–435.
24. A. L. Rebenko, *High and low temperature expansions in the classical statistical mechanics of continuous systems* (Marseille, 1998), Mathematical Results in Statistical Mechanics (Eds. S. Miracle-Sole', J. Ruiz, and V. Zagrebnov), World Sci. Publishing, Singapore, 1999, pp. 110–119.
25. A. L. Rebenko, *A new proof of Ruelle's superstability bounds*, J. Stat. Phys. **91** (1998), no. 3–4, 815–826.
26. A. L. Rebenko, *Cell gas model of classical statistical systems*, Rev. Math. Phys. **25** (2013), no. 4, 1330006 (28 pages).
27. D. Ruelle, *Cluster property of the correlation functions of classical gases*, Rev. Mod. Phys. **36** (1964), 580–584.
28. D. Ruelle, *Statistical Mechanics: Rigorous results*, W. A. Benjamin, Inc., New York—Amsterdam, 1969.
29. J. L. Silva, Yu. G. Kondratiev, and L. Streit, *Representation of diffeomorphisms on compound Poisson space* (Marseille, 1997), Analysis on Infinite-Dimensional Lie Groups and Algebras (Eds. H. Heyer, J. Marion), World Sci. Publishing, New Jersey, 1998, pp. 376–393.
30. C. Y. Shen, *A functional calculus approach to the Ursell-Mayer functions*, J. Math. Phys. **13** (1972), 754–759.
31. C. Y. Shen, *On a certain class of transformations in statistical mechanics*, J. Math. Phys. **14** (1973), 1202–1204.

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