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STRONG BASE FOR FUZZY TOPOLOGY

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Abstract. It is known that a base for a traditional topology, or for a L-topology, τ, is a subset $\mathcal{B}$ of τ with the property that every element $G \in \tau$ can be written as a union of elements of $\mathcal{B}$. In the classical case it is equivalent to say that $G \in \tau$ if and only if for any $x \in G$ we have $B \in \mathcal{B}$ satisfying $x \in B \subseteq G$. This latter property is taken as the foundation for a notion of strong base for a L-topology. Characteristic properties of a strong base are given and among other results it is shown that a strong base is a base, but not conversely.

It is well known that a base for a traditional topology, or for a L-topology, τ, is a subset $\mathcal{B}$ of τ with the property that every element $G \in \tau$ can be written as a union of elements of $\mathcal{B}$. In the classical case, but not in the L-topology case, it is equivalent to say that $G \in \tau$ if and only if for any $x \in G$ we have $B \in \mathcal{B}$ satisfying $x \in B \subseteq G$. This latter property is taken as the foundation for a notion of strong base for a L-topology.

The two conditions

(i) each point is contained in at least one member of the base, and
(ii) if a point belongs to the intersection of two members of the base, then it belongs to a member of the base contained in that intersection,

which are characteristic of a base of a traditional topology, are shown to be insufficient to characterize a strong base for a L-topology. Therefore, characteristic properties of a strong base are given and among other results it is shown that a strong base is a base, but not conversely.

Throughout this paper, L will be a Hutton algebra, i.e. complete and completely distributive lattice which has an order-reversing involution $' : L \to L$, a smallest element 0 and a largest element 1 ($0 \neq 1$). Obviously, for every set X, L^X, the family of all L-subsets of X, i.e., all mappings from X to L, is also a Hutton algebra under the pointwise order and induced order-reversing involution; we denote the largest element and the smallest element of L^X by 1_X and 0_X, respectively. A *L-topological space*, briefly, *L-ts*, is a pair (X, δ), where X is a set and δ, called an *L-topology* on X, is a subfamily of L^X which contains 0_X and 1_X, and is closed under the operations of taking finite intersections and arbitrary unions. Obviously, in case $L = [0,1]$, a L-topological space $([0,1]^X, \delta)$ (for simplicity, denoted by (X, δ)) is just a fuzzy topological space in the sense of Chang [1].

We assume that the reader is familiar with the usual notions and basic concepts of L-topology and lattice theory.

For $x \in X$ and $\alpha \in L$ ($\alpha \neq 0$) denote the L-subset taking value α at x and value 0 at other points of X by x_α, call it an *L-point* on X. The family of all L-points in X is denoted by $\mathcal{F}$ (see [3], [6]). A L-point x_α is said to belong to A, written $x_\alpha \in A$, where A is an L-subset in X, iff $\alpha \leq A(x)$. For all undefined basic concepts, our reference is [2–5].

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Definition 1. A subset $\mathcal{B}$ of a L-topology δ on X which has the property

$$U \in \delta \iff (x_\alpha \in U \;\Rightarrow\; \exists\, B \in \mathcal{B} \text{ with } x_\alpha \in B \subseteq U)$$

is called a **strong base** of δ.

Theorem 1. *A subset $\mathcal{B}$ of L^X is a strong base for some L-topology on X if and only if it satisfies the following three conditions:*
(i) $x_\alpha \in \mathcal{F} \;\Rightarrow\; \exists\, B_{x_\alpha} \in \mathcal{B}$ *with* $x_\alpha \in B_{x_\alpha}$,
(ii) $B_1, B_2 \in \mathcal{B}$ *and* $x_\alpha \in B_1 \wedge B_2 \Rightarrow \exists\, B_3 \in \mathcal{B}$ *with* $x_\alpha \in B_3 \subset (B_1 \wedge B_2)$,
(iii) $\{B_\lambda\}_{\lambda \in J} \subset \mathcal{B}$ *and* $x_\alpha \in \bigvee_\lambda B_\lambda \Rightarrow \exists B \in \mathcal{B}$ *with* $x_\alpha \in B \subset \bigvee_\lambda B_\lambda$.

Proof. It is clear that if $\mathcal{B}$ is a strong base for some L-topology on X then it satisfies the conditions (i)–(iii). Conversely, suppose that for subset $\mathcal{B}$ of L^X the conditions (i)–(iii) are satisfied. We shall show that the set

$$\delta_{\mathcal{B}} = \{U \subset L^X \mid x_\alpha \in U \;\Rightarrow\; \exists\, B_{x_\alpha} \in \mathcal{B} \textit{ with } x_\alpha \in B_{x_\alpha} \subseteq U\}$$

is a L-topology (so-called *L-topology generated by* $\mathcal{B}$) on X.

$0_X \in \delta_{\mathcal{B}}$ is trivial. For every $x_\alpha \in \mathcal{F}$ there exists $B_{x_\alpha} \in \mathcal{B}$ satisfying $x_\alpha \in B_{x_\alpha}$ by (i). Since $B_{x_\alpha} \subset \mathbf{1}_X$ we have $x_\alpha \in B_{x_\alpha} \subset \mathbf{1}_X$, so $\mathbf{1}_X \in \delta_{\mathcal{B}}$.

Let $\{U_\lambda\}_{\lambda \in J} \subset \delta_{\mathcal{B}}$ and $x_\alpha \in \bigvee_\lambda U_\lambda$. Let $\alpha_\lambda := U_\lambda(x)$. Then $x_\alpha \in \bigvee_\lambda x_{\alpha_\lambda}$. Since $U_\lambda \in \delta_{\mathcal{B}}$, then $\exists\, B_\lambda \in \mathcal{B}$ with $x_{\alpha_\lambda} \in B_\lambda \subset U_\lambda$. Hence

$$x_\alpha \in \bigvee_\lambda x_{\alpha_\lambda} \subset \bigvee_\lambda B_\lambda \subset \bigvee_\lambda U_\lambda.$$

By (iii) there exists an element B of $\mathcal{B}$ such that $x_\alpha \in B \subset \bigvee_\lambda B_\lambda$. Therefore $x_\alpha \in B \subset \bigvee_\lambda U_\lambda$. Thus $\bigvee_\lambda U_\lambda \in \delta_{\mathcal{B}}$.

Let $U, V \in \delta_{\mathcal{B}}$ and $x_\alpha \in U \wedge V$. Then $x_\alpha \in U$ and $x_\alpha \in V$. Therefore $\exists\, B_1, B_2 \in \mathcal{B}$ with $x_\alpha \in B_1 \subset U$ and $x_\alpha \in B_2 \subset V$. Hence $x_\alpha \in B_1 \wedge B_2 \subset U \wedge V$. From (ii) there is an element $B_3 \in \mathcal{B}$ such that $x_\alpha \in B_3 \subset B_1 \wedge B_2$. Then $x_\alpha \in B_3 \subset U \wedge V$, i.e. $U \wedge V \in \delta_{\mathcal{B}}$. ☐

Example 1. . Let $X = \{a, b\}$ and $L = [0, 1]$. Let $\mathcal{B}$ be the collection of the following L-subsets: $B_0 = (x, \underset{4/5}{a}, \underset{1/2}{b})$, $B_1 = (x, \underset{1}{a}, \underset{1/2}{b})$, $B_2 = (x, \underset{1/2}{a}, \underset{1}{b})$, $B_3 = (x, \underset{1/2}{a}, \underset{1/2}{b})$, $B_n = (x, \underset{4/5-1/n}{a}, \underset{1/2}{b})$ $(n \geq 4)$.

It is easy to check that for $\mathcal{B}$ the conditions (i), (ii) and (iii) of Theorem 1 are valid. Therefore, $\mathcal{B}$ is a strong base for some L-topology on X.

Theorem 2. *Every strong base of a L-topology is a base.*

Proof. Let $\mathcal{B}$ be a strong base on X and $\mathcal{A}_{\mathcal{B}}$ the subset of all unions of elements of $\mathcal{B}$. We shall show that the subset $\mathcal{A}_{\mathcal{B}}$ is a L-topology on X.

The first condition of L-topology is trivial. If $\{U_\lambda\}_{\lambda \in J} \subset \mathcal{A}_{\mathcal{B}}$, then for every λ there exists a family $\{B_{\mu,\lambda}\} \subset \mathcal{B}$ with $U_\lambda = \bigvee_\mu B_{\mu,\lambda}$. Hence $\bigvee_\lambda U_\lambda = \bigvee_{\mu,\lambda} B_{\mu,\lambda} \in \mathcal{A}_{\mathcal{B}}$, i.e. the second condition of L-topology is also true.

Put $U = \bigvee_\lambda B_\lambda, V = \bigvee_\mu B_\mu \in \mathcal{A}_{\mathcal{B}}$ and let $x_\alpha \in U \wedge V = \big(\bigvee_\lambda B_\lambda\big) \wedge \big(\bigvee_\mu B_\mu\big)$. Then $x_\alpha \in \bigvee_\lambda B_\lambda$ and $x_\alpha \in \bigvee_\mu B_\mu$. By (iii) there are $B, B' \in \mathcal{B}$ such that

$$x_\alpha \in B \subset \bigvee_\lambda B_\lambda \quad \text{and} \quad x_\alpha \in B' \subset \bigvee_\mu B_\mu.$$

Hence

$$x_\alpha \in B \wedge B' \subset \Big(\bigvee_\lambda B_\lambda\Big) \wedge \Big(\bigvee_\mu B_\mu\Big).$$

There exists by (ii) an element $B_{x,\alpha}$ of $\mathcal{B}$ with $x_\alpha \in B_{x,\alpha} \subset B \wedge B'$. Therefore $x_\alpha \in B_{x,\alpha} \subset \left(\bigvee_\lambda B_\lambda\right) \wedge \left(\bigvee_\mu B_\mu\right)$ and $U \wedge V = \left(\bigvee_\lambda B_\lambda\right) \wedge \left(\bigvee_\mu B_\mu\right) = \bigvee_{x,\alpha} B_{x,\alpha}$, i.e. $U \wedge V \in \mathcal{A}_{\mathcal{B}}$. Thus $\mathcal{A}_{\mathcal{B}}$ is a L-topology on X. □

Remark 1. The converse of Theorem 2 is false:

Example 2. Consider the family $\mathcal{B}$ of Example 1 without the set B_0.

It is easy to check that for $\mathcal{B}$ the conditions (i) and (ii) of Theorem 1 are valid again and it is base for some L-topology on X. But, now for $\mathcal{B}$ the condition (iii) is not true. For this we consider the set $\bigvee_{n\geq 4} B_n = (x, \underset{4/5}{a}, \underset{1/2}{b})$.

It is easy to see that for this set there is no set $B \in \mathcal{B}$ with $B(a) \geq \alpha$ and $B \subset \bigvee_{n\geq 4} B_n$. Hence, for the L-point a_α there does not exist a set $B \in \mathcal{B}$ with $a_\alpha \in B \subset \bigvee_{n\geq 4} B_n$. Therefore, $\bigvee_{n\geq 4} B_n \notin \delta_{\mathcal{B}}$, i.e. $\delta_{\mathcal{B}}$ is not a L-topology on X. Since condition (iii) of Theorem 1 is not satisfied, $\mathcal{B}$ is not a strong base.

Now, we consider other example.

Example 3. Let $(I[L], \delta)$ be the L-fuzzy unit interval with its canonical L-topology. We briefly recall the definitions from [2], [3].

Let $md_{\mathbb{R}}(L)$ be the family of all the monotonically decreasing mappings $\lambda \in L^{\mathbb{R}}$ with $\bigvee_{t\in\mathbb{R}} \lambda(t) = 1$ and $\bigwedge_{t\in\mathbb{R}} \lambda(t) = 0$. Let $md_I(L)$ be the family of all the elements in $md_{\mathbb{R}}(L)$ with $\lambda(t) = 1$ for $t < 0$ and $\lambda(t) = 0$ for $t > 1$. Define an equivalence relation $\sim$ on $md_I(L)$ as: $\lambda \sim \mu \Leftrightarrow \forall\ t,\ \lambda(t-) = \mu(t-),\ \lambda(t+) = \mu(t+)$, where $\lambda(t-) = \bigwedge_{s<t} \lambda(s)$, $\lambda(t+) = \bigvee_{s>t} \lambda(s)$. Denote the family of all the equivalence classes in $md_I(L)$ with respect to $\sim$ by $I[L]$. For every $t \in \mathbb{R}$, define $L_t, R_t \in L^{I[L]}$ as: $L_t(\lambda) = \lambda(t-)'$ and $R_t(\lambda) = \lambda(t+)$.

Let $\mathcal{B}_L^I$ be the set of all finite intersections of elements of $\mathcal{S}_L^I = \{L_t, R_t \in L^{I[L]} : t \in \mathbb{R}\}$.

It is easy to show that the set $\mathcal{B}_L^I$ is a base for a L-topology, which we denote by δ. Moreover, for the set $\mathcal{B}_L^I$ conditions (i) and (ii) of Theorem 1 are immediately satisfied, because $R_t = \mathbf{1}_X$ for $t < 0$; $L_s = \mathbf{1}_X$ for $s > 1$, and $\mathcal{B}_L^I$ is closed under finite intersections.

Let us to show that for the set $\mathcal{B}_L^I$ the condition (iii) is not true. Let $\lambda : \mathbb{R} \to [0, 1]$ be the continuous function defined by

$$\lambda(x) := \begin{cases} 1 & \text{if} \quad x < 0, \\ 1 - x & \text{if} \quad 0 \leq x \leq 1, \\ 0 & \text{if} \quad x > 1 \end{cases}$$

and for $n \geq 2$ consider points $t_n,\ s_n \in \mathbb{R}$ defined by

$$t_n := \begin{cases} \frac{1}{2} + 2^{-n} & \text{for } n \text{ even}, \\ \frac{1}{4} + 2^{-n} & \text{for } n \text{ odd} \end{cases} \quad \text{and} \quad s_n := \begin{cases} \frac{3}{4} - 2^{-n} & \text{for } n \text{ even}, \\ \frac{1}{2} - 2^{-n} & \text{for } n \text{ odd}. \end{cases}$$

A straightforward calculation shows that $L_{s_n}(\lambda) \wedge R_{t_n}(\lambda) = \frac{1}{2} - 2^{-n}$ for all $n \geq 2$, so $\bigvee_{n=2}^{\infty}(R_{t_n} \wedge L_{s_n})(\lambda) = \frac{1}{2}$. Hence, for the L-point $\lambda_{1/2}$ we have $\lambda_{1/2} \in \bigvee_{n=2}^{\infty}(R_{t_n} \wedge L_{s_n})$. Now, suppose that there exists an element $R_{t_o} \wedge L_{s_o} \in \mathcal{B}_L^I$ with

$$\lambda_{1/2} \in R_{t_o} \wedge L_{s_o} \subset \bigvee_{n=2}^{\infty} (R_{t_n} \wedge L_{s_n}).$$

Since $(R_{t_o} \wedge L_{s_o})(\lambda) = 1/2$, we have $t_o \leq \frac{1}{2}$, $s_o \geq \frac{1}{2}$ and at least one of the its is equal to $1/2$. Let, for simplicity, $t_o = \frac{1}{2}$ and $s_o \geq \frac{1}{2}$. We define the function μ as

$$\mu(x) := \begin{cases} 1 & \text{if} \quad x \leq 3/4, \\ 1/2 & \text{if} \quad 3/4 < x \leq 1, \\ 0 & \text{if} \quad x > 1. \end{cases}$$

A straightforward calculation shows that

$$R_{1/2}(\mu) = \bigvee_{t>1/2} \mu(t) = 1 \quad \text{and} \quad L_{s_o}(\mu) = 1 - \bigwedge_{s<s_0} \mu(s) = 1 - 0 = 1,$$

so $\big(R_{1/2} \wedge L_{s_o}\big)(\mu) = 1$. Similarly,

$$R_{t_n}(\mu) = \bigvee_{t>t_n} \mu(t) = 1 \quad \text{and} \quad L_{s_n}(\mu) = 1 - \bigwedge_{s<s_n} \mu(s) = 1 - 1 = 0,$$

so $\bigvee_{n=2}^{\infty}(R_{t_n} \wedge L_{s_n})(\mu) = 0$. This is a contradiction.

Therefore, for the L-point $\lambda_{1/2}$ there does not exist a set $R_{t_o} \wedge L_{s_o} \in \mathcal{B}_L^I$ with $\lambda_{1/2} \in R_{t_o} \wedge L_{s_o} \subset \bigvee_{n=2}^{\infty}(R_{t_n} \wedge L_{s_n})$. Thus, for the set $\mathcal{B}_L^I$ the condition (iii) is not true. By Theorem 1, $\mathcal{B}$ is not a strong base for δ.

Remark 2. In traditional topology condition (iii) of Theorem 1 is automatically satisfied since,

$\{B_\lambda\}_{\lambda \in J} \subset \mathcal{B}$ and $x \in \bigcup_\lambda B_\lambda \;\Rightarrow\; \exists\ \lambda_0 \in J$ such that $x \in B_{\lambda_0} \subset \bigcup_\lambda B_\lambda$.

Thus, in this case the notions of base and strong base coincide.

As noted above if $\mathcal{B}$ is a strong base then the subset $\mathcal{A}_\mathcal{B}$ is a L-topology. On the other hand, by definition the subset $\delta_\mathcal{B}$ also is a L-topology. In the following theorem we'll show that these topologies coincide.

Theorem 3. *If $\mathcal{B}$ is a strong base then $\mathcal{A}_\mathcal{B} = \delta_\mathcal{B}$.*

Proof. Since $\{B_\lambda\} \subset \mathcal{B}$ and $\bigvee_\lambda B_\lambda \in \delta_\mathcal{B}$ we have $\mathcal{A}_\mathcal{B} \subset \delta_\mathcal{B}$. Conversely, let $U \in \delta_\mathcal{B}$. Choose, for each $x_\alpha \in U$, an element B_x of $\mathcal{B}$ such that $x_\alpha \in B_x \subset U$. Then $U = \bigvee_x B_x$, so U equals a union of elements of $\mathcal{B}$, i.e. $U \in \mathcal{A}_\mathcal{B}$. $\square$

Remark 3. According to Example 2, in order for $\delta_\mathcal{B}$ to be a L-topology it is necessary that $\mathcal{B}$ satisfy condition (iii). On the other hand, conditions (i) and (ii) are sufficient for $\mathcal{A}_\mathcal{B}$ to be a L-topology.

Clearly the first and second conditions of L-topology are satisfied. To check the third condition, we consider

$$\Big(\bigvee_\lambda B_\lambda\Big) \wedge \Big(\bigvee_\mu B_\mu\Big) = \bigvee_{\lambda,\mu}(B_\lambda \wedge B_\mu).$$

If $x_\alpha \in (B_\lambda \wedge B_\mu)$ there is, by (ii), an element $B_{\alpha,x,\lambda,\mu}$ of $\mathcal{B}$ such that $x_\alpha \in B_{\alpha,x,\lambda,\mu} \subset (B_\lambda \wedge B_\mu)$. Hence $B_\lambda \wedge B_\mu = \bigvee_{\alpha,x} B_{\alpha,x,\lambda,\mu}$. Therefore,

$$\Big(\bigvee_\lambda B_\lambda\Big) \wedge \Big(\bigvee_\mu B_\mu\Big) = \bigvee_{\alpha,x,\lambda,\mu} B_{\alpha,x,\lambda,\mu},$$

i.e. $\big(\bigvee_\lambda B_\lambda\big) \wedge \big(\bigvee_\mu B_\mu\big) \in \mathcal{A}_\mathcal{B}$. Thus $\mathcal{A}_\mathcal{B}$ is a L-topology.

Sometimes we need to go in the reverse direction, from a L-topology to a strong base that generates it. Below it is one way of obtaining a strong base for a given topology.

Theorem 4. *Let (X,δ) be a L-ts. Suppose that $\mathcal{C}$ is a subset of elements of δ such that for each $U \in \delta$ and $x_\alpha \in U$, there is an element C_x of $\mathcal{C}$ such that $x_\alpha \in C_x \subset U$. Then $\mathcal{C}$ is a strong base for δ.*

Proof. Immediate from Definition 1. □

Remark 4. By Remark 3, a subset $\mathcal{B}$ of L^X with the properties (i), (ii) and without the condition (iii), generates the L-topology $\mathcal{A}_\mathcal{B}$. Theorem 4 shows that δ is a strong base of itself.

The following theorem shows when one L-topology is finer than another in terms of strong bases for these topologies.

Theorem 5. *Let $\mathcal{B}$ and $\mathcal{B}'$ be strong bases for the L-topologies δ ve δ' on X, respectively. Then the followings are equivalent:*

(1) $\delta \subseteq \delta'$.
(2) *For each $x_\alpha \in \mathcal{F}$ and each base member $B \in \mathcal{B}$ containing x_α, there is a base member $B' \in \mathcal{B}'$ such that $x_\alpha \in B' \subset B$.*

Proof. $\Longleftarrow$ Let $U \in \delta$ and $x_\alpha \in U$. Since $\mathcal{B}$ generates δ, there is a base member $B \in \mathcal{B}$ with $x_\alpha \in B \subset U$. By hypothesis there is a base member $B' \in \mathcal{B}'$ with $x_\alpha \in B' \subset B$. Hence $x_\alpha \in B' \subset U$, so $U \in \delta'$, by definition.

$\Longrightarrow$ We are given $x_\alpha \in \mathcal{F}$ and $B \in \mathcal{B}$ with $B \ni x_\alpha$. B belongs to δ by definition and $\delta \preceq \delta'$ by hypothesis; therefore $B \in \delta'$. Since δ' is generated by $\mathcal{B}'$, there is a base member $B' \in \mathcal{B}'$ such that $x_\alpha \in B' \subset B$. □

As an application of Theorem 5 we give the following example.

Example 4. Let $\mathcal{B}$ be the set of all *fuzzy circular regions* in the plane $\mathbb{R}^2$, i.e. the set of all fuzzy sets of the form $B(A^\beta_{(x,y)}, r)$, where $r > 0$, $A^\beta_{(x,y)} \in \mathcal{F}$ and

$$B(A^\beta_{(x,y)}, r)(A^\alpha_{(x',y')}) := \begin{cases} \alpha\beta, & \text{if} \quad \sqrt{(x-x')^2+(y-y')^2} + |\alpha-\beta| < r, \\ 0, & \text{otherwise,} \end{cases}$$

in other words $B(A^\beta_{(x,y)}, r) := \bigcup\limits_{\substack{\sqrt{(x-x')^2+(y-y')^2} \\ +|\alpha-\beta|<r}} A^{\alpha\beta}_{(x',y')}$.

Similarly, let $\mathcal{C}$ be the collection of all *fuzzy rectangular regions* in the plane $\mathbb{R}^2$, i.e. the collection of all fuzzy sets of the form $C(A^\beta_{(x,y)}, a)$, where $a > 0$, $A^\beta_{(x',y')} \in \mathcal{F}$ and

$$C(A^\beta_{(x,y)}, a)(A^\alpha_{(x',y')}) := \begin{cases} \alpha\beta, & \text{if} \quad \max\{|x-x'|, |y-y'|\} + |\alpha-\beta| < a, \\ 0, & \text{otherwise} \end{cases}$$

for each $A^\alpha_{(x',y')} \in \mathcal{F}$.

It is easy to see that the sets $\mathcal{B}$ and $\mathcal{C}$ are strong bases of L-topologies on $\mathbb{R}^2$ and by Theorem 5 the set $\mathcal{B}$ generates the same L-topology as the set $\mathcal{C}$.

Let us, for instance, to show that the set $\mathcal{B}$ satisfies the condition (iii) of Theorem 1. Let $A^\alpha_{(x,y)}$ be a L-point and $\{B_\lambda\}_{\lambda\in\Lambda} \subset \mathcal{B}$ with $A^\alpha_{(x,y)} \in \bigvee_{\lambda\in\Lambda} B_\lambda$. We may suppose without loss of generality that $B_\lambda = B(A^{\beta_\lambda}_{(x,y)}, r_\lambda)$ for any $\lambda \in \Lambda$. Then one has $\sup_\lambda \beta_\lambda = 1$. If $\alpha = 1$ we choose the set $B \in \mathcal{B}$ as $B(A^1_{(x,y)}, r)$, for some $r \in \{r_\lambda\}_{\lambda\in\Lambda}$. Let $\alpha < 1$ and $r = \sup_\lambda r_\lambda$. Then there exists $\lambda_0 \in \Lambda$ with $\alpha < \beta_{\lambda_0}$. If $r < \infty$, we choose the set $B \in \mathcal{B}$ as $B(A^{\beta_{\lambda_0}}_{(x,y)}, r)$; if $r = \infty$, as $B(A^{\beta_{\lambda_0}}_{(x,y)}, r')$, where $r' = 2|\alpha - \beta_{\lambda_0}|$. It is all one implies that $A^\alpha_{(x,y)} \in B \subset \bigvee_{\lambda\in\Lambda} B(A^{\beta_\lambda}_{(x,y)}, r_\lambda)$. Thus, for the set $\mathcal{B}$ the condition (iii) is true.

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