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A NOTE ON EQUILIBRIUM GLAUBER AND KAWASAKI DYNAMICS FOR PERMANENTAL POINT PROCESSES

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Abstract. We construct two types of equilibrium dynamics of an infinite particle system in a locally compact metric space X for which a permanental point process is a symmetrizing, and hence invariant measure. The Glauber dynamics is a birth-and-death process in X, while in the Kawasaki dynamics interacting particles randomly hop over X. In the case $X = \mathbb{R}^d$, we consider a diffusion approximation for the Kawasaki dynamics at the level of Dirichlet forms. This leads us to an equilibrium dynamics of interacting Brownian particles for which a permanental point process is a symmetrizing measure.

1. Introduction

Let X be a locally compact Polish space and let ν be a Radon non-atomic measure on it. Let $\Gamma = \Gamma_X$ denote the space of all locally finite subsets (configurations) in X.

A Glauber dynamics (a birth-and-death process of an infinite system of particles in X) is a Markov process on Γ whose formal (pre-)generator has the form

$$\begin{aligned}(L_{\mathrm{G}}F)(\gamma) &= \sum_{x\in\gamma} d(x,\gamma\setminus x)(F(\gamma\setminus x)-F(\gamma))\\ &\quad+\int_X \nu(dx)\, b(x,\gamma)\,(F(\gamma\cup x)-F(\gamma))\,,\qquad \gamma\in\Gamma.\end{aligned}\tag{1.1}$$

Here and below, for simplicity of notation we write x instead of $\{x\}$. The coefficient $d(x, \gamma \setminus x)$ describes the rate at which particle x of configuration γ dies, while $b(x, \gamma)$ describes the rate at which, given configuration γ, a new particle is born at x.

A Kawasaki dynamics (a dynamics of hopping particles) is a Markov process on Γ whose formal (pre-)generator is

$$(L_{\mathrm{K}}F)(\gamma) = \sum_{x\in\gamma} c(x,y,\gamma\setminus x)(F(\gamma\setminus x\cup y)-F(\gamma)),\quad \gamma\in\Gamma.\tag{1.2}$$

The coefficient $c(x, y, \gamma \setminus x)$ describes the rate at which particle x of configuration γ hops to y, taking the rest of the configuration, $\gamma \setminus x$, into account.

Equilibrium Glauber and Kawasaki dynamics which have a standard Gibbs measure as symmetrizing (and hence invariant) measure were constructed in [19, 20]. In [22], this construction was extended to the case of an equilibrium dynamics which has a determinantal (fermion) point process as invariant measure, For further studies of equilibrium and non-equilibrium Glauber and Kawasaki dynamics, we refer to [3, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 21, 28] and the references therein.

The aim of this note is to show that general criteria of existence of Glauber and Kawasaki dynamics which were developed in [22] are applicable to a wide class of α-permanental ($\alpha \in \mathbb{N}$) point processes, proposed by Shirai and Takahashi [30]. This

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class includes classical permanental (boson) point processes, see e.g. [5, 30]. We will also consider a diffusion approximation for the Kawasaki dynamics at the level of Dirichlet forms (compare with [15]). This will lead us to an equilibrium dynamics of interacting Brownian particles for which an α-permanental point process is a symmetrizing measure. As a by-product of our considerations, we will also extend the result of [30] on the existence of α-permanental point process.

2. Equilibrium Glauber and Kawasaki dynamics – general results

Let X be a locally compact Polish space. We denote by $\mathcal{B}(X)$ the Borel σ-algebra on X, and by $\mathcal{B}_0(X)$ the collection of all sets from $\mathcal{B}(X)$ which are relatively compact. We fix a Radon, non-atomic measure on $(X, \mathcal{B}(X))$. (For most applications, the reader may think of X as $\mathbb{R}^d$ and ν as the Lebegue measure.)

The configuration space Γ over X is defined as the set of all subsets of X which are locally finite

$$\Gamma := \big\{\gamma \subset X : |\gamma_\Lambda| < \infty \text{ for each } \Lambda \in \mathcal{B}_0(X)\big\},$$

where $|\cdot|$ denotes the cardinality of a set and $\gamma_\Lambda := \gamma \cap \Lambda$. One can identify any $\gamma \in \Gamma$ with the positive Radon measure $\sum_{x\in\gamma} \varepsilon_x$, where ε_x is the Dirac measure with mass at x and $\sum_{x\in\varnothing} \varepsilon_x$:=zero measure. The space Γ can be endowed with the vague topology, i.e., the weakest topology on Γ with respect to which all maps

$$\Gamma \ni \gamma \mapsto \langle \varphi, \gamma\rangle := \int_X \varphi(x)\,\gamma(dx) = \sum_{x\in\gamma} \varphi(x), \quad \varphi \in C_0(X),$$

are continuous. Here, $C_0(X)$ is the space of all continuous, real-valued functions on X with compact support. We denote the Borel σ-algebra on Γ by $\mathcal{B}(\Gamma)$. A point process in X is a probability measure on $(\Gamma, \mathcal{B}(\Gamma))$.

We fix a point process μ which satisfies the so-called condition (Σ'_ν) [5, 26], i.e., there exist a measurable function $r : X \times \Gamma \to [0, +\infty]$, called the Papangelou intensity of μ, such that

$$\int_\Gamma \mu(d\gamma) \int_X \gamma(dx) F(x, \gamma) = \int_\Gamma \mu(d\gamma) \int_X \nu(dx)\, r(x, \gamma) F(x, \gamma \cup x) \tag{2.1}$$

for any measurable function $F : X \times \Gamma \to [0, +\infty]$. The condition (Σ'_ν) can be thought of as a kind of weak Gibbsianess of μ. Intuitively, we may treat the Papangelou intensity as

$$r(x, \gamma) = \exp[-E(x, \gamma)], \tag{2.2}$$

where $E(x, \gamma)$ is the relative energy of interaction between particle x and configuration γ.

To define an equilibrium Glauber dynamics for which μ is a symmetrizing measure, we fix a death coefficient as a measurable function $d : X \times \Gamma \to [0, +\infty]$, and then define a birth coefficient $b : X \times \Gamma \to [0, +\infty]$ by

$$b(x, \gamma) = d(x, \gamma) r(x, \gamma), \quad (x, \gamma) \in X \times \Gamma. \tag{2.3}$$

To define a Kawasaki dynamics, we fix a measurable function $c : X^2 \times \Gamma^2 \to [0, +\infty]$ which satisfies

$$r(x, \gamma) c(x, y, \gamma) = r(y, \gamma) c(y, x, \gamma), \quad (x, y, \gamma) \in X^2 \times \Gamma. \tag{2.4}$$

Formulas (2.3) and (2.4) are called the balance conditions [13, 14]. We will also assume that the function $c(x, y, \gamma)$ vanishes if at least one of the functions $r(x, \gamma)$ and $r(y, \gamma)$ vanishes, i.e.,

$$c(x, y, \gamma) = c(x, y, \gamma) \chi_{\{r>0\}}(x, \gamma) \chi_{\{r>0\}}(y, \gamma). \tag{2.5}$$

Here, for a set A, χ_A denotes the indicator function of A. We refer to [22, Remark 3.1] for a justification of this assumption, which involves the interpretation of $r(x,\gamma)$ as in (2.2), see also Remark 2.4 below.

We denote by $\mathcal{F}C_{\mathrm{b}}(C_0(X),\Gamma)$ the space of all functions of the form

$$\Gamma \ni \gamma \mapsto F(\gamma) = g(\langle \varphi_1, \gamma\rangle, \dots, \langle \varphi_N, \gamma\rangle), \tag{2.6}$$

where $N \in \mathbb{N}$, $\varphi_1, \dots, \varphi_N \in C_0(X)$ and $g \in C_{\mathrm{b}}(\mathbb{R}^N)$. Here, $C_{\mathrm{b}}(\mathbb{R}^N)$ denotes the space of all continuous bounded functions on $\mathbb{R}^N$. We assume that, for each $\Lambda \in \mathcal{B}_0(X)$,

$$\int_\Gamma \mu(d\gamma) \int_\Lambda \gamma(dx)\, d(x, \gamma\setminus x) < \infty, \tag{2.7}$$

$$\int_\Gamma \mu(d\gamma) \int_X \gamma(dx) \int_X \nu(dy)\, c(x,y,\gamma\setminus x)(\chi_\Lambda(x) + \chi_\Lambda(y)) < \infty. \tag{2.8}$$

As easily seen, conditions (2.7) and (2.8) are sufficient in order to define bilinear forms

$$\begin{aligned}
\mathcal{E}_{\mathrm{G}}(F,G) :&= \int_\Gamma \mu(d\gamma) \int_X \gamma(dx)\, d(x,\gamma\setminus x)(F(\gamma\setminus x) - F(\gamma))(G(\gamma\setminus x) - G(\gamma)),\\
\mathcal{E}_{\mathrm{K}}(F,G) :&= \frac{1}{2}\int_\Gamma \mu(d\gamma) \int_X \gamma(dx) \int_X \nu(dy)\, c(x,y,\gamma\setminus x)(F(\gamma\setminus x\cup y) - F(\gamma))\\
&\quad \times (G(\gamma\setminus x\cup y) - G(\gamma)),
\end{aligned}$$

where $F, G \in \mathcal{F}C_{\mathrm{b}}(C_0(X),\Gamma)$.

For the construction of the Kawasaki dynamics, we will also assume that the following technical assumptions holds:

$$\begin{aligned}
&\exists u, v \in \mathbb{R} \quad \forall \Lambda \in \mathcal{B}_0(X):\\
&\int_\Lambda \gamma(dx) \int_\Lambda \nu(dy)\, r(x,\gamma\setminus x)^u r(y,\gamma\setminus x)^v c(x,y,\gamma\setminus y) \in L^2(\Gamma,\mu) < \infty.
\end{aligned} \tag{2.9}$$

Note that in formula (2.9) and below, we use the convention $\frac{0}{0} := 0$.

The following theorem was essentially proved in [22].

Theorem 2.1. *(i) Assume that a point process μ satisfies* (2.1). *Assume that conditions* (2.3), (2.7), *respectively* (2.4), (2.5), (2.8), *and* (2.9) *are satisfied. Let $\sharp = \mathrm{G}, \mathrm{K}$. Then the bilinear form $(\mathcal{E}_\sharp, \mathcal{F}C_{\mathrm{b}}(C_0(x),\Gamma))$ is closable in $L^2(\Gamma,\mu)$ and its closure will be denoted by $(\mathcal{E}_\sharp, D(\mathcal{E}_\sharp))$. Further there exists a conservative Hunt process (Glauber, respectively Kawasaki dynamics)*

$$M^\sharp = \Big(\Omega^\sharp, \mathcal{F}^\sharp, (\mathcal{F}^\sharp_t)_{t\ge0}, (\Theta^\sharp_t)_{t\ge0}, (X^\sharp(t))_{t\ge0}, (P^\sharp_\gamma)_{\gamma\in\Gamma}\Big)$$

on Γ which is properly associated with $(\mathcal{E}_\sharp, D(\mathcal{E}_\sharp))$, i.e., for all ($\mu$-version of) $F \in L^2(\Gamma,\mu)$ and $t > 0$

$$\Gamma \ni \gamma \mapsto p^\sharp_t F(\gamma) := \int_{\Omega^\sharp} F(X^\sharp(t))\, dP^\sharp_\gamma$$

is an $\mathcal{E}^\sharp$-quasi continuous version of $\exp(tL_\sharp)F$, where $(-L_\sharp, D(L_\sharp))$ is the generator of $(\mathcal{E}_\sharp, D(\mathcal{E}_\sharp))$. $M^\sharp$ is up-to μ-equivalence unique. In particular, $M^\sharp$ is μ-symmetric and has μ as invariant measure.

(ii) $M^\sharp$ from (i) is up to μ-equivalence unique between all Hunt processes

$$M' = \big(\Omega', \mathcal{F}', (\mathcal{F}'_t)_{t\ge0}, (\Theta'_t)_{t\ge0}, (X'(t))_{t\ge0}, (P'_\gamma)_{\gamma\in\Gamma}\big)$$

on Γ having μ as invariant measure and solving a martingale problem for $(L_\sharp, D(L_\sharp))$, i.e., for all $G \in D(H_\sharp)$

$$\widetilde{G}(X'(t)) - \widetilde{G}(X'(0)) - \int_0^t (L_\sharp G)(X'(s))\, ds, \quad t \ge 0,$$

is an $(\mathcal{F}'_t)$*-martingale under* P'_γ *for* $\mathcal{E}_\sharp$*-q.e.* $\gamma \in \Gamma$*. Here,* $\widetilde{G}$ *denotes an* $\mathcal{E}_\sharp$*-quasi-continuous version of* G*.*

(iii) Further assume that, for each $\Lambda \in \mathcal{B}_0(X)$*,*

$$\int_\Lambda \gamma(dx)\, d(x, \gamma \setminus x) \in L^2(\Gamma, \mu), \quad \int_\Lambda \nu(dx)\, b(x, \gamma) \in L^2(\Gamma, \mu), \tag{2.10}$$

in the Glauber case, and

$$\int_X \gamma(dx) \int_X \nu(dy)\, c(x, y, \gamma \setminus x)(\chi_\Lambda(x) + \chi_\Lambda(y)) \in L^2(\Gamma, \mu) \tag{2.11}$$

in the Kawasaki case. Then $\mathcal{F}C_\mathrm{b}(C_0(X), \Gamma) \subset D(L_\sharp)$*, and for each* $F \in \mathcal{F}C_\mathrm{b}(C_0(X), \Gamma)$*,* $L_\sharp F$ *is given by formulas* (1.1) *and* (1.2)*, respectively.*

Remark 2.1. We refer to [24] for an explanation of notions appearing in Theorem 2.1, see also a brief explanation of them in [22].

Proof of Theorem 2.1. The statement follows from Theorems 3.1 and 3.2 in [22]. Note that, although these theorems are formulated for determinantal point processes only, their proof only uses the (Σ'_ν) property of these point processes. Note also that condition (2.9) is formulated in [22] only for $v = 1$, however the proof of Lemma 3.2 in [22] admits a straightforward generalization to the case of an arbitrary $v \in \mathbb{R}$. □

Remark 2.2. Part (iii) of Theorem 2.1 states that the operator $(-L_\sharp, D(L_\sharp))$ is the Friedrichs' extension of the operator $(-L_\sharp, \mathcal{F}C_\mathrm{b}(C_0(X), \Gamma))$ defined by formulas (1.1), (1.2), respectively.

Let us fix a parameter $s \in [0, 1]$ and define

$$d(x, \gamma) : = r(x, \gamma)^{s-1} \chi_{\{r>0\}}(x, \gamma), \quad (x, \gamma) \in X \times \Gamma, \tag{2.12}$$

$$b(x, \gamma) : = r(x, \gamma)^{s} \chi_{\{r>0\}}(x, \gamma), \quad (x, \gamma) \in X \times \Gamma, \tag{2.13}$$

$$\begin{aligned} c(x, y, \gamma) := a(x, y) r(x, \gamma)^{s-1} r(y, \gamma)^s \chi_{\{r>0\}}(x, \gamma) \chi_{\{r>0\}}(y, \gamma), \\ (x, y, \gamma) \in X^2 \times \Gamma. \end{aligned} \tag{2.14}$$

Here the function $a : X^2 \to [0, +\infty)$ is bounded, measurable, symmetric (i.e., $a(x, y) = a(y, x)$), and satisfies

$$\sup_{x \in X} \int_X a(x, y)\, \nu(dy) < \infty. \tag{2.15}$$

Note that the balance conditions (2.3) and (2.4) are satisfied for these coefficients, and so is condition (2.5).

Remark 2.3. Note that, if $X = \mathbb{R}^d$ and $a(x, y)$ has the form $a(x - y)$ for a function $a : \mathbb{R}^d \to [0, \infty)$, then condition (2.15) means that $a \in L^1(\mathbb{R}^d, dx)$. (Here and below, in the case $X = \mathbb{R}^d$, we use an obvious abuse of notation.)

Remark 2.4. Using representation (2.2), we can rewrite formulas (2.12)–(2.14) as follows:

$$\begin{aligned} d(x, \gamma \setminus x) &= \exp[(1 - s) E(x, \gamma \setminus x)] \chi_{\{E<+\infty\}}(x, \gamma \setminus x), \\ b(x, \gamma \setminus x) &= \exp[-s E(x, \gamma \setminus x)] \chi_{\{E<+\infty\}}(x, \gamma \setminus x), \\ c(x, y, \gamma \setminus x) &= a(x, y) \exp[(1 - s) E(x, \gamma \setminus x) - s E(y, \gamma \setminus x)] \\ &\quad \times \chi_{\{E<+\infty\}}(x, \gamma \setminus x) \chi_{\{E<+\infty\}}(y, \gamma \setminus x). \end{aligned}$$

So, if the corresponding dynamics exist, one can give the following heuristic description of them: Both dynamics are concentrated on configurations $\gamma \in \Gamma$ such that, for each $x \in \gamma$, the relative energy of interaction between x and the rest of configuration, $\gamma \setminus x$, is finite; those particles tend to die, respectively hop, which have a high energy of interaction with the rest of the configuration, while it is more probable that a new particle is born at y, respectively x hops to y, if the energy of interaction between y and the rest of the configuration is low.

Let us assume that the point process μ satisfies:

$$\forall \Lambda \in \mathcal{B}_0(X): \quad \int_\Lambda \gamma(dx) \in L^2(\Gamma, \mu).$$

Then, by choosing $u = 1 - s$ and $v = -s$ in (2.9), we conclude that the coefficient c given by (2.14) satisfies (2.9).

We will construct below a class of point processes μ for which the coefficients d, b and c given above satisfy the other conditions of Theorem 2.1.

3. Permanental point processes and corresponding equilibrium dynamics

Let K be a linear, bounded, self-adjoint operator on the real space $L^2(X, \nu)$. Further assume that $K \geq 0$ and K is locally of trace class, i.e., $\mathrm{Tr}(P_\Lambda K P_\Lambda) < \infty$ for all $\Lambda \in \mathcal{B}_0(X)$, where P_Λ denotes the operator of multiplication by χ_Λ. Hence, each operator $P_\Lambda \sqrt{K}$ is of Hilbert–Schmidt class. Following [23] (see also [12, Lemma A.4]), we conclude that $\sqrt{K}$ is an integral operator whose integral kernel, $\varkappa(x, y)$, satisfies

$$\int_\Lambda \int_X \nu(dx)\nu(dy)\varkappa(x,y)^2 < \infty \quad \text{for all} \quad \Lambda \in \mathcal{B}_0(X). \tag{3.1}$$

In particular,

$$\varkappa(x, \cdot) \in L^2(X, \nu) \quad \text{for } \nu\text{-a.a.} \quad x \in X. \tag{3.2}$$

Hence, K is an integral operator whose integral kernel can be chosen as

$$\begin{aligned} k(x,y) &= \int_X \varkappa(x,z)\varkappa(z,y)\nu(dz) \\ &= \int_X \varkappa(x,z)\varkappa(y,z)\nu(dz) = (\varkappa(x,\cdot), \varkappa(y,\cdot))_{L^2(X,\nu)}. \end{aligned} \tag{3.3}$$

We also have, for each $\Lambda \in \mathcal{B}_0(X)$,

$$\begin{aligned} \mathrm{Tr}(P_\Lambda K P_\Lambda) &= \|\sqrt{K} P_\Lambda\|_{\mathrm{HS}}^2 \\ &= \int_\Lambda \nu(dx) \int_X \nu(dy)\varkappa(x,y)^2 = \int_\Lambda k(x,x)\,\nu(dx), \end{aligned} \tag{3.4}$$

where $\|\cdot\|_{\mathrm{HS}}$ denotes the Hilbert–Schmidt norm.

Proposition 3.1. *There exists a random field $(Y(x))_{x \in X}$ on a probability space $(\Omega, \mathcal{A}, P)$ such that the mapping*

$$X \times \Omega \ni (x, \omega) \mapsto Y(x, \omega) \tag{3.5}$$

is measurable, and for ν-a.a. $x \in X$, $Y(x)$ is a Gaussian random variable with mean 0 and such that

$$\mathbb{E}\,(Y(x)Y(y)) = k(x,y) \quad \textit{for } \nu^{\otimes 2}\textit{-a.a. } (x,y) \in X^2 \textit{ and } \nu\textit{-a.a. } x = y \in X. \tag{3.6}$$

Remark 3.1. The statement of Proposition 3.1 is well-known if the integral kernel of the operator K admits a continuous version (see e.g. Theorem 1.8 and p. 456 in [30]). In the latter case, $(Y(x))_{x \in X}$ is a Gaussian random field and formula (3.6) holds for all $(x, y) \in X^2$.

Proof of Proposition 3.1. Consider a standard triple of real Hilbert spaces

$$H_+ \subset H_0 = L^2(X,\nu) \subset H_- .$$

Here the Hilbert space H_+ is densely and continuously embedded into H_0, the inclusion operator $H_+ \hookrightarrow H_0$ is of Hilbert–Schmidt class, and the Hilbert space H_- is the dual space of H_+ with respect to the center space H_0 (see e.g. [2]).

Let $\mathbb{P}$ be the standard Gaussian measure on H_-, i.e., the probability measure on the Borel σ-algebra $\mathcal{B}(H_-)$ which has Fourier transform

$$\int_{H_-} e^{i\langle\omega,f\rangle}\,\mathbb{P}(d\omega) = \exp\Big[-\frac{1}{2}\|f\|^2_{H_0}\Big], \quad f\in H_+ ,$$

where $\langle\omega,f\rangle$ denotes the dual pairing between $\omega\in H_-$ and $f\in H_+$. Then the mapping $H_+\ni f\to\langle\cdot,f\rangle$ can be extended by continuity to an isometry

$$I : H_0 \to L^2(H_-,\mathbb{P}). \tag{3.7}$$

For any $f\in H_0$ we denote $\langle\cdot,f\rangle := If$. Thus, for each $f\in H_0$, $\langle\cdot,f\rangle$ is a (complex) Gaussian random variable with mean 0 and for any $f,g\in H_0$

$$\int_{H_-}\langle\omega,f\rangle\langle\omega,g\rangle\,\mathbb{P}(d\omega) = (f,g)_{L^2(X,\nu)}. \tag{3.8}$$

Thus, by (3.2), we set for ν-a.a. $x\in X$, $\widetilde{Y}(x,\omega) := \langle\omega,k(x,\cdot)\rangle$. Hence $\widetilde{Y}(x)$ is a Gaussian random variable and by (3.3) and (3.8), (3.6) holds.

Hence, it remains to prove that there exists a random field $Y=(Y(x))_{x\in X}$ for which the mapping (3.5) is measurable and such that $Y(x,\omega)=\widetilde{Y}(x,\omega)$ for $\nu\otimes\mathbb{P}$-a.a. (x,ω). To this end, we fix any $\Lambda\in\mathcal{B}_0(X)$ and denote by $\mathcal{B}(\Lambda)$ the trace σ-algebra of $\mathcal{B}(X)$ on Λ. We define a set $\mathcal{D}_\Lambda$ of the functions $u:\Lambda\times X\to\mathbb{R}$ of the form

$$u(x,y) = \sum_{i=1}^n \chi_{\Delta_i}(x) f_i(y), \tag{3.9}$$

where $\Delta_i\in\mathcal{B}(\Lambda)$, $f_i\in H_+$, $i=1,\dots,n$. Define a linear mapping

$$I_\Lambda : \mathcal{D}_\Lambda \to L^2(\Lambda\times H_-,\nu\otimes\mathbb{P}) \tag{3.10}$$

by setting, for each $u\in\mathcal{D}_\Lambda$ of the form (3.9),

$$(I_\Lambda u)(x,\omega) = \sum_{i=1}^n \chi_{\Delta_i}(x)\langle\omega,f_i\rangle, \quad (x,\omega)\in\Lambda\times H_- .$$

Clearly, I_Λ can be extended to an isometry

$$I_\Lambda : L^2(\Lambda\times X,\nu^{\otimes 2}) \to L^2(\Lambda\times H_-,\nu\otimes\mathbb{P}),$$

and we have $I_\Lambda = \mathbf{1}_\Lambda\otimes I$, where $\mathbf{1}_\Lambda$ is the identity operator in $L^2(\Lambda,\nu)$ and the operator I is as in (3.7).

Fix any $u\in L^2(\Lambda\times X,\nu^{\otimes 2})$. As easily seen, there exist a sequence $(u_n)_{n=1}^\infty\subset\mathcal{D}_\Lambda$ such that $u_n\to u$ in $L^2(\Lambda\times X,\nu^{\otimes 2})$ and for ν-a.a. $x\in\Lambda$, $u_n(x,\cdot)\to u(x,\cdot)$ in $L^2(X,\nu)$ Hence, for ν-a.a. $x\in\Lambda$, $I_\Lambda u_n(x,\cdot)\to I_\Lambda u(x,\cdot)$ in $L^2(H_-,\mathbb{P})$, which implies

$$(I_\Lambda u)(x,\omega) = \langle\omega,u(x,\cdot)\rangle \quad \text{for } \mathbb{P}\text{-a.a. } \omega\in H_- . \tag{3.11}$$

Now, denote by $\varkappa_\Lambda$ the restriction of $\varkappa$ to the set $\Lambda\times X$. For ν-a.a. $x\in\Lambda$, we define $Y_\Lambda(x) := (I_\Lambda\varkappa_\Lambda)(x,\cdot)$. Hence, by (3.11), for ν-a.a. $x\in\Lambda$, $Y_\Lambda(x)=\widetilde{Y}(x)$ $\mathbb{P}$-a.e. Finally, let $(\Lambda_n)_{n=1}^\infty\subset\mathcal{B}_0(X)$ be such that $\Lambda_n\cap\Lambda_m=\emptyset$ if $n\neq m$ and $\bigcup_{n=1}^\infty\Lambda_n = X$. Setting $Y(x) := Y_{\Lambda_n}(x)$ for ν-a.a. $x\in\Lambda_n, n\in\mathbb{N}$, we conclude the statement. $\square$

Let Y be a random field as in Proposition 3.1. For each $\Lambda \in \mathcal{B}_0(X)$, we have

$$\begin{aligned}\mathbb{E}\left(\int_\Lambda Y(x)^2\,\nu(dx)\right) &= \int_\Lambda \mathbb{E}(Y(x)^2)\,\nu(dx)\\ &= \int_\Lambda \nu(dx)\int_X \nu(dy)\varkappa(x,y)^2 < \infty.\end{aligned}$$

In particular, the function $Y(x)^2$ is locally ν-integrable $\mathbb{P}$-a.s. Let $l \in \mathbb{N}$ and let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space on which l independent copies $Y_1, Y_2, \dots, Y_l$ of a random field Y as in Proposition 3.1 are defined. Denote by $\mu^{(l)}$ the Cox point process on X with random intensity $g^{(l)}(x) = \sum_{i=1}^l Y_i(x)^2$, which is locally ν-integrable $\mathbb{P}$-a.s. Thus, $\mu^{(l)}$ is the probability measure on $(\Gamma, \mathcal{B}(\Gamma))$ which satisfies

$$\int_\Gamma \mu^{(l)}(d\gamma)F(\gamma) = \int_\Omega \mathbb{P}(d\omega)\int_\Gamma \pi_{g^{(l)}(x,\omega)\nu(dx)}(d\gamma)F(\gamma) \tag{3.12}$$

for each measurable function $F : \Gamma \to [0, +\infty]$. Here, for a locally ν-integrable function $g : X \to [0, +\infty)$, we denote by $\pi_{g(x)\nu(dx)}$ the Poisson point process in X with intensity measure $g(x)\nu(dx)$, see e.g [5]. This is the unique point process in X which satisfies the Mecke identity

$$\int_\Gamma \pi_{g(x)\nu(dx)}(d\gamma)\int_X \gamma(dx)F(x,\gamma) = \int_\Gamma \pi_{g(x)\nu(dx)}(d\gamma)\int_X \nu(dx)\, g(x)F(x,\gamma\cup x) \tag{3.13}$$

for each measurable $F : X \times \Gamma \to [0, +\infty]$. By (3.12) and (3.13) (compare with e.g. [27]), for each $l \in \mathbb{N}$, the point process $\mu^{(l)}$ satisfies condition (Σ'_ν) and its Papangelou intensity is given by

$$r^{(l)}(x,\gamma) = \widetilde{\mathbb{E}}(g^{(l)}(x) \mid \mathcal{F})(\gamma) = \widetilde{\mathbb{E}}\Big(\sum_{i=1}^l Y_i(x)^2 \mid \mathcal{F}\Big)(\gamma). \tag{3.14}$$

Here $\widetilde{\mathbb{E}}$ denotes the (conditional) expectation with respect to the probability measure

$$\widetilde{\mathbb{P}}(d\omega, d\gamma) = \widetilde{\mathbb{P}}(d\omega)\,\pi_{g^{(l)}(x,\omega)\nu(dx)}(d\gamma) \tag{3.15}$$

on $\Omega \times \Gamma$, while $\mathcal{F}$ denotes the σ-algebra on $\Omega \times \Gamma$ generated by the mappings

$$\Omega \times \Gamma \ni (\omega, \gamma) \to F(\gamma) \in \mathbb{R},$$

where $F : \Gamma \to \mathbb{R}$ is measurable.

Recall that a point process μ in X is said to have correlation functions if, for each $n \in \mathbb{N}$, there exist a non-negative, measurable, symmetric function $k_\mu^{(n)}$ on X^n such that, for any measurable, symmetric function $f^n : X^n \to [0, +\infty]$,

$$\begin{aligned}&\int_\Gamma \sum_{\{x_1,\dots,x_n\}\subset\gamma} f^{(n)}(x_1,\dots,x_n)\,\mu(d\gamma)\\ &\quad= \frac{1}{n!}\int_{X^n} f^{(n)}(x_1,\dots,x_n)k_\mu^{(n)}(x_1,\dots,x_n)\nu(dx_1)\cdots\nu(dx_n).\end{aligned} \tag{3.16}$$

As well known (e.g. [5]), for a locally ν-integrable function $g : X \to [0, +\infty)$, the Poisson point process $\pi_{g(x)\nu(dx)}$ has correlation functions

$$k_\mu^{(n)}(x_1,\dots,x_n) = g(x_1)\cdots g(x_n). \tag{3.17}$$

Let us recall the notion of α-permanent [31], called α-determinant in [30]. For a square matrix $A = (a_{ij})_{i,j=1}^n$ and $\alpha \in \mathbb{R}$, we set

$$\operatorname{per}_\alpha A := \sum_{\sigma\in S_n} \alpha^{n-m(\sigma)}\prod_{i=1}^n a_{i\sigma(i)},$$

where S_n is the group of all permutations of $\{1,\dots,n\}$ and $m(\sigma)$ denotes the number of cycles in σ. In particular, $\operatorname{per}_1 A$ is the usual permanent of A, while $\operatorname{per}_{-1} A$ is the usual determinant of A. Analogously to [30, subsec. 6.4], we conclude from (3.12), (3.16) and (3.17) that the point process $\mu^{(l)}$ has correlation functions

$$k^{(n)}_{\mu^{(l)}}(x_1,\dots,x_n)=\operatorname{per}_{\frac{l}{2}}(lk(x_i,x_j))_{i,j=1}^n \quad \text{for } \nu^{\otimes n}\text{-a.a. } (x_1,\dots,x_n)\in X^n. \tag{3.18}$$

For $l=2$, the point process $\mu^{(2)}$ is often called a boson point process, see e.g. [5, 23]. Thus, we have proved the following

Proposition 3.2. *For each $l\in\mathbb N$, there exists a point process $\mu^{(l)}$ in X whose correlation functions are given by* (3.18). *The $\mu^{(l)}$ satisfies condition* (Σ'_ν) *and its Papangelou intensity is given by* (3.14).

Remark 3.2. Recall that in [30], under the same assumptions on the operator K, the existence of a point process with correlation functions (3.18) was proved for even $l\in\mathbb N$, and for odd $l\in\mathbb N$ the statement of Proposition 3.2 was proved under the additional assumption of continuity of the integral kernel $k(\cdot,\cdot)$.

We will now prove that, for a point process $\mu^{(l)}$ as in Proposition 3.2, Glauber and Kawasaki dynamics with coefficients (2.12), (2.13) and (2.14), respectively exist.

Theorem 3.1. *(i) For each point process $\mu^{(l)}$ as in Proposition 3.2, the coefficients $d(x,\gamma)$ and $b(x,\gamma)$ defined by* (2.12) *and* (2.13), *satisfy conditions* (2.3) *and* (2.7) *and so statements (i) and (ii) of Theorem 2.1 hold, in particular, a corresponding Glauber dynamics exists.*

(ii) Assume additionally that $k(x,x)$ is bounded outside a set $\Delta\in\mathcal B_0(X)$. Then for a point process $\mu^{(l)}$ as in Proposition 3.2, the coefficient $c(x,y,\gamma)$ defined by (2.14), *satisfies* (2.4), (2.5), (2.8) *and* (2.9), *and so statements (i) and (ii) of Theorem 2.1 hold, in particular, a corresponding Kawasaki dynamics exists.*

Proof. We start with the following

Lemma 3.1. *For each $n\in\mathbb N$ and for ν-a.a. $x\in X$*

$$\int_\Gamma r(x,\gamma)^n\,\mu(d\gamma)\le\frac{(2n)!}{2^n\,n!}\,k(x,x)^n. \tag{3.19}$$

Proof. Using Jensen's inequality for conditional expectation and the formula for moments of a Gaussian measure (see e.g. [2, Chapter 2, Section 2, Lemma 2.1]), we have

$$\begin{aligned}\int_\Gamma r(x,\gamma)^n\,\mu(d\gamma)&=\widetilde{\mathbb E}(\widetilde{\mathbb E}(Y(x)^2\mid\mathcal F)^n)\le\widetilde{\mathbb E}(\widetilde{\mathbb E}(Y(x)^{2n}\mid\mathcal F))\\&=\widetilde{\mathbb E}(Y(x)^{2n})\le\frac{(2n)!}{2^n\,n!}\,\|\varkappa(x,\cdot)\|^{2n}_{L^2(X,\nu)}=\frac{(2n)!}{2^n\,n!}\,k(x,x)^n\end{aligned}$$

for ν-a.a. $x\in X$. □

We will only prove statement (ii) of Theorem 3.1, as the proof of statement (i) is similar and simper. Also, for simplicity of notation, we will only consider the case $l=1$ (for $l>1$ the proof being similar). We will also omit the upper index (1) from our notation. By (2.1) we have, for each $\Lambda\in\mathcal B_0(X)$,

$$\begin{aligned}&\int_\Gamma\mu(d\gamma)\int_X\gamma(dx)\int_X\nu(dy)\,c(x,y,\gamma\setminus x)(\chi_\Lambda(x)+\chi_\Lambda(y))\\&\quad=\int_\Gamma\mu(d\gamma)\int_X\nu(dx)\int_X\nu(dy)\,r(x,\gamma)c(x,y,\gamma)(\chi_\Lambda(x)+\chi_\Lambda(y))\\&\quad=\int_\Gamma\mu(d\gamma)\int_X\nu(dx)\int_X\nu(dy)\,a(x,y)r(x,\gamma)^s r(y,\gamma)^s\chi_{\{r>0\}}(x,\gamma)\end{aligned} \tag{3.20}$$

$$
\begin{aligned}
&\times \chi_{\{r>0\}}(y,\gamma)(\chi_\Lambda(x)+\chi_\Lambda(y))\\
&\le \int_\Gamma \mu(d\gamma)\int_X \nu(dx)\int_X \nu(dy)\, a(x,y) r(x,\gamma)^s r(y,\gamma)^s(\chi_\Lambda(x)+\chi_\Lambda(y))\\
&= 2\int_\Gamma \mu(d\gamma)\int_\Lambda \nu(dx)\int_X \nu(dy)\, a(x,y) r(x,\gamma)^s r(y,\gamma)^s\\
&\le 2\int_\Gamma \mu(d\gamma)\int_\Lambda \nu(dx)\int_X \nu(dy)\, a(x,y)(1+r(x,\gamma))(1+r(y,\gamma)).
\end{aligned}
$$

By (2.15)

$$
\int_\Gamma \mu(d\gamma)\int_\Lambda \nu(dx)\int_X \nu(dy)\, a(x,y)<\infty. \tag{3.21}
$$

Below, $C_i, i=1,2,3,\dots,$ will denote positive constants whose explicit values are not important for us. We have, by (2.15)

$$
\begin{aligned}
&\int_\Gamma \mu(d\gamma)\int_\Lambda \nu(dx)\int_X \nu(dy)\, a(x,y) r(x,\gamma)\\
&\quad= \int_\Gamma \mu(d\gamma)\int_\Lambda \nu(dx)\, r(x,\gamma)\Big(\int_X \nu(dy)\, a(x,y)\Big)\\
&\quad\le C_1\int_\Gamma \mu(d\gamma)\int_\Lambda \nu(dx) r(x,\gamma)\\
&\quad= C_1\int_\Gamma \mu(d\gamma)\int_\Lambda \gamma(dx)=C_1\int_\Lambda k(x,x)\,\nu(dx)<\infty.
\end{aligned}\tag{3.22}
$$

Next, by (3.14)

$$
\begin{aligned}
&\int_\Gamma \mu(d\gamma)\int_\Lambda \nu(dx)\int_X \nu(dy)\, a(x,y) r(y,\gamma)\\
&\quad= \int_\Lambda \nu(dx)\int_X \nu(dy)\, a(x,y)\int_\Gamma \mu(d\gamma) r(y,\gamma)\\
&\quad= \int_\Lambda \nu(dx)\int_X \nu(dy)\, a(x,y) k(y,y)\\
&\quad= \int_\Lambda \nu(dx)\int_\Delta \nu(dy)\, a(x,y) k(y,y)+\int_\Lambda \nu(dx)\int_{\Delta^c} \nu(dy)\, a(x,y) k(y,y)\\
&\quad\le C_2\int_\Lambda \nu(dx)\int_\Delta \nu(dy) k(y,y)+C_3\int_\Lambda \nu(dx)\int_{\Delta^c} \nu(dy)\, a(x,y)<\infty,
\end{aligned}\tag{3.23}
$$

where we used that the function a is bounded and $k(y,y)$ is bounded on Δ^c. Analogously, using Lemma 3.1, we have

$$
\begin{aligned}
&\int_\Gamma \mu(d\gamma)\int_\Lambda \nu(dx)\int_X \nu(dy) a(x,y) r(x,\gamma) r(y,\gamma)\\
&\quad\le \int_\Lambda \nu(dx)\int_X \nu(dy)\, a(x,y)\|r(x,\cdot)\|_{L^2(\mu)}\,\|r(y,\cdot)\|_{L^2(\mu)}\\
&\quad\le C_4\int_\Lambda \nu(dx)\int_X \nu(dy)\, a(x,y) k(x,x) k(y,y)\\
&\quad\le C_5\int_\Lambda \nu(dx)\, k(x,x)\int_\Delta \nu(dy)\, k(y,y)\\
&\quad+ C_6\int_\Lambda \nu(dx) k(x,x)\int_{\Delta^c} \nu(dy)\, a(x,y)<\infty.
\end{aligned}\tag{3.24}
$$

Thus, by (3.20)–(3.24), the theorem is proved. $\square$

Theorem 3.2. *(i) Let $s \in \left[\frac{1}{2}, 1\right]$, and let the conditions of Theorem 3.1 (i) be satisfied. Then the coefficients $d(x,\gamma)$ and $b(x,\gamma)$ defined by* (2.12) *and* (2.13)*, satisfy condition* (2.10)*. Thus, $\mathcal{F}C_{\mathrm{b}}(C_0(X),\Gamma) \subset D(L_{\mathrm{G}})$, and for each $F \in \mathcal{F}C_{\mathrm{b}}(C_0(X),\Gamma)$, $L_{\mathrm{G}}F$ is given by formula* (1.1).

(ii) Let $s \in \left[\frac{1}{2}, 1\right]$, and let the conditions of Theorem 3.1 (ii) be satisfied. Further assume that either

$$\forall \Lambda \in \mathcal{B}_0(X)\ \exists \Lambda' \in \mathcal{B}_0(X)\ \forall x \in \Lambda\ \forall y \in (\Lambda')^c: \quad a(x,y) = 0, \tag{3.25}$$

or

$$\int_\Delta k(x,x)^2\, \nu(dx) < \infty, \tag{3.26}$$

where Δ is as in Theorem 3.1 (ii). Then the coefficient $c(x,y,\gamma)$ defined by (2.14)*, satisfies condition* (2.11)*. Thus, $\mathcal{F}C_{\mathrm{b}}(C_0(X),\Gamma) \subset D(L_{\mathrm{K}})$, and for each $F \in \mathcal{F}C_{\mathrm{b}}(C_0(X),\Gamma)$, $L_{\mathrm{K}}F$ is given by formula* (1.2).

Remark 3.3. If $X = \mathbb{R}^d$ and the function a is as in Remark 2.3, then condition (3.25) means that the function $\tilde{a}$ has a compact support.

Proof of Theorem 3.2. We again prove only the part related to Kawasaki dynamics and only in the case $l = 1$, omitting the upper index (1) from our notation. We first assume that (3.25) is satisfied. Since the function a is bounded and satisfies (3.25), it suffices to show that, for each $\Lambda \in \mathcal{B}_0(X)$,

$$\int_\Lambda \gamma(dx) \int_\Lambda \nu(dy) r(x, \gamma\setminus x)^{s-1} r(y, \gamma\setminus x)^s \chi_{\{r>0\}}(x, \gamma\setminus x) \chi_{\{r>0\}}(y, \gamma\setminus x) \in L^2(\mu). \tag{3.27}$$

We note that, for $s \in \left[\frac{1}{2}, 1\right]$, $2s-1 \in [0,1]$. Therefore, by the Cauchy inequality, we have

$$\begin{aligned}
&\int_\Gamma \mu(d\gamma)\Big(\int_\Lambda \gamma(dx)\, r(x, \gamma\setminus x)^{s-1} \chi_{\{r>0\}}(x, \gamma\setminus x)\\
&\quad \times \int_\Lambda \nu(dy)\, r(y, \gamma\setminus x)^s \chi_{\{r>0\}}(y, \gamma\setminus x)\Big)^2\\
&\quad \le \int_\Gamma \mu(d\gamma) \int_\Lambda \gamma(dx)\, r(x, \gamma\setminus x)^{2(s-1)} \chi_{\{r>0\}}(x, \gamma\setminus x)\\
&\quad \times \Big(\int_\Lambda \nu(dy)\, r(y, \gamma\setminus x)^s \chi_{\{r>0\}}(y, \gamma\setminus x)\Big)^2 \gamma(\Lambda)\\
&\quad = \int_\Gamma \mu(d\gamma) \int_\Lambda \nu(dx)\, r(x,\gamma)^{2s-1} \chi_{\{r>0\}}(x,\gamma)\\
&\quad \times \Big(\int_\Lambda \nu(dy) r(y,\gamma)^s \chi_{\{r>0\}}(y,\gamma)\Big)^2 (\gamma(\Lambda)+1)\\
&\quad \le \int_\Gamma \mu(d\gamma) \Big(\int_\Lambda \nu(dx)(1 + r(x,\gamma))\Big)^3 (\gamma(\Lambda)+1)\\
&\quad \le \Big(\int_\Gamma \mu(d\gamma)\Big(\int_\Lambda \nu(dx)(1+r(x,\gamma))\Big)^6\Big)^{1/2} \Big(\int_\Gamma \mu(d\gamma)(\gamma(\Lambda)+1)^2\Big)^{1/2}.
\end{aligned} \tag{3.28}$$

By Lemma 3.1, we have, for each $n \in \mathbb{N}$,

$$\begin{aligned}
&\int_\Gamma \mu(d\gamma)\Big(\int_\Lambda \nu(dx)\, r(x,\gamma)\Big)^n = \int_\Lambda \nu(dx_1) \cdots \int_\Lambda \nu(dx_n) \int_\Gamma \mu(d\gamma)\, r(x_1,\gamma)\cdots r(x_n,\gamma)\\
&\quad \le \int_\Lambda \nu(dx_1) \cdots \int_\Lambda \nu(dx_n) \|r(x_1,\cdot)\|_{L^n(\mu)} \cdots \|r(x_n,\cdot)\|_{L^n(\mu)}\\
&\quad \le \frac{(2n)!}{2^n n!}\Big(\int_\Lambda \nu(dx) k(x,x)\Big)^n < \infty
\end{aligned} \tag{3.29}$$

Now, (3.27) follows from (3.28) and (3.29).

Next, we assume that (3.26) is satisfied. We fix $\Lambda \in \mathcal{B}_0(X)$ and denote

$$u(x,y) := a(x,y)(\chi_\Lambda(x) + \chi_\Lambda(x)).$$

Then, by the Cauchy inequality,

$$\begin{aligned}
&\int_\Gamma \mu(d\gamma)\Big(\int_X \gamma(dx)\int_X \nu(dy)\, u(x,y) r(x,\gamma\setminus x)^{s-1}\chi_{\{r>0\}}(x,\gamma\setminus x)\\
&\times r(y,\gamma\setminus x)^s \chi_{\{r>0\}}(y,\gamma\setminus x)\Big)^2\\
&\le \int_\Gamma \mu(d\gamma)\int_X \gamma(dx)\int_X \nu(dy)\, u(x,y) r(x,\gamma\setminus x)^{2(s-1)}\chi_{\{r>0\}}(x,\gamma\setminus x)\\
&\times r(y,\gamma\setminus x)^{2s}\chi_{\{r>0\}}(y,\gamma\setminus x)\int_X \gamma(dx')\int_X \nu(dy')\, u(x',y')\\
&= \int_\Gamma \mu(d\gamma)\int_X \nu(dx)\int_X \nu(dy)\, u(x,y) r(x,\gamma)^{2s-1}\chi_{\{r>0\}}(x,\gamma)\\
&\times r(y,\gamma)^{2s}\chi_{\{r>0\}}(y,\gamma)\int_X (\gamma+\varepsilon_x)(dx')\int_X \nu(dy')\, u(x',y')\\
&\le \int_\Gamma \mu(d\gamma)\int_X \nu(dx)\int_X \nu(dy)\, u(x,y)(1+r(x,\gamma))(1+r(y,\gamma)^2\\
&\times\Big(\int_X \gamma(dx')\int_X \nu(dy')u(x',y') + \int_X \nu(dy')u(x,y')\Big).
\end{aligned}$$

By (2.15), it suffices to prove that

$$\int_\Gamma \mu(d\gamma)\Big(\int_X \nu(dx)\int_X \nu(dy)\, u(x,y)(1+r(x,\gamma))(1+r(y,\gamma)^2)\Big)^2 < \infty, \tag{3.30}$$

$$\int_\Gamma \mu(d\gamma)\Big(\int_X \gamma(dx)\int_X \nu(dy)\, u(x,y)\Big)^2 < \infty. \tag{3.31}$$

We first to prove (3.31). We have, by Proposition 3.2,

$$\begin{aligned}
&\int_\Gamma \Big(\int_X \gamma(dx)\int_X \nu(dy) u(x,y)\Big)^2\\
&= \int_X \nu(dy)\int_X \nu(dy')\int_\Gamma \mu(d\gamma)\int_X \gamma(dx)\int_X \gamma(dx')\, u(x,y)u(x',y')\\
&= \int_X \nu(dy)\int_X \nu(dy')\int_\Gamma \mu(d\gamma)\Big(\int_X \gamma(dx)\, u(x,y)u(x,y')\\
&+ \int_X \gamma(dx)\int_X (\gamma-\varepsilon_x)(dx')\, u(x,y)u(x',y')\Big)\\
&= \int_X \nu(dy)\int_X \nu(dy')\Big(\int_X \nu(dx)\, k(x,x)u(x,y)u(x,y')\\
&+ \int_X \nu(dx)\int_X \nu(dx')\Big(\frac{1}{2}\, k(x,x')^2 + k(x,x)k(x',x')\Big)u(x,y)u(x',y')\Big)\\
&\le \int_X \nu(dy)\int_X \nu(dy')\Big(\int_X \nu(dx)\, k(x,x)u(x,y)u(x,y')\\
&+ \int_X \nu(dx)\int_X \nu(dx')\,\frac{3}{2}\, k(x,x)k(x',x')u(x,y)u(x',y')\Big)\\
&= \int_X \nu(dy)\int_X \nu(dy')\int_X \nu(dx)\, k(x,x)u(x,y)u(x,y')
\end{aligned}$$

$$
\begin{aligned}
&+\frac{3}{2}\Big(\int_X \nu(dy)\int_X \nu(dx)\,k(x,x)u(x,y)\Big)^2\\
&\le \int_\Delta \nu(dx)\,k(x,x)\Big(\int_X \nu(dy)\,u(x,y)\Big)^2\\
&+C_7\int_X \nu(dy)\int_X \nu(dy')\int_X \nu(dx)\,u(x,y)u(x,y')\\
&+\frac{3}{2}\Big(\int_\Delta \nu(dx)\,k(x,x)\int_X \nu(dy)\,u(x,y)+C_7\int_X \nu(dy)\int_X \nu(dx)\,u(x,y)\Big)^2<\infty.
\end{aligned}
$$

Next, we prove (3.30). By Lemma 3.1 and (3.26), we have

$$
\begin{aligned}
&\int_\Gamma \mu(d\gamma)\Big(\int_X \nu(dx)\int_X \nu(dy)\,u(x,y)(1+r(x,\gamma))(1+r(y,\gamma)^2)\Big)^2\\
&\quad=\int_X \nu(dx)\int_X \nu(dx')\int_X \nu(dy)\int_X \nu(dy')\,u(x,y)u(x',y')\\
&\quad\times\int_\Gamma \mu(d\gamma)(1+r(x,\gamma))(1+r(x',\gamma))(1+r(y,\gamma)^2)(1+r(y',\gamma)^2)\\
&\quad\le\int_X \nu(dx)\int_X \nu(dx')\int_X \nu(dy)\int_X \nu(dy')\,u(x,y)u(x',y')\left(1+\|r(x,\cdot)\|_{L^4(\mu)}\right)\\
&\quad\times\left(1+\|r(x',\cdot)\|_{L^4(\mu)}\right)\left(1+\|r(y,\cdot)^2\|_{L^4(\mu)}\right)\left(1+\|r(y',\cdot)^2\|_{L^4(\mu)}\right)\\
&\quad\le C_8\Big(\int_X \nu(dx)\int_X \nu(dy)\,u(x,y)(1+k(x,x))(1+k(y,y)^2)\Big)^2<\infty.
\end{aligned}
$$

Thus, the theorem is proved. □

4. Diffusion approximation

¿From now on, we set $X=\mathbb{R}^d$, $d\in\mathbb{N}$, and ν to be Lebesgue measure. We will show that, under an appropriate scaling, the Dirichlet form of the Kawasaki dynamics converges to a Dirichlet form which identifies a diffusion process on Γ having a permanental point process $\mu^{(l)}$ as a symmetrizing measure. The way we scale the Kawasaki dynamics will be similar to the ansatz of [15].

We denote by $\mathcal{F}C_{\mathrm{b}}^\infty(C_0^\infty(\mathbb{R}^d),\Gamma)$ the space of all functions of the form (2.6) where $N\in\mathbb{N}$, $\varphi_1,\dots,\varphi_N\in C_0^\infty(\mathbb{R}^d)$ and $g\in C_{\mathrm{b}}^\infty(\mathbb{R}^N)$. Here, $C_0^\infty(\mathbb{R}^d)$ denotes the space of smooth functions on $\mathbb{R}^d$ with compact support, and $C_{\mathrm{b}}^\infty(\mathbb{R}^N)$ denotes the space of all smooth bounded functions on $\mathbb{R}^N$ whose all derivatives are bounded. Clearly,

$$\mathcal{F}C_{\mathrm{b}}^\infty(C_0^\infty(\mathbb{R}^d),\Gamma)\subset\mathcal{F}C_{\mathrm{b}}(C_0(\mathbb{R}^d),\Gamma),$$

and the set $\mathcal{F}C_{\mathrm{b}}^\infty(C_0^\infty(\mathbb{R}^d),\Gamma)$ is a core for the Dirichlet form $(\mathcal{E}_{\mathrm{K}},D(\mathcal{E}_{\mathrm{K}}))$.

We fix $s=1/2$. Let us assume that the function $a(x,y)$ is as in Remark 2.3. Thus, the coefficient $c(x,y,\gamma)$ has the form

$$c(x,y,\gamma)=a(x-y)r(x,\gamma)^{-1/2}r(y,\gamma)^{1/2}\chi_{\{r>0\}}(x,\gamma)\chi_{\{r>0\}}(y,\gamma). \tag{4.1}$$

Note that $y-x$ describes the change of the position of a particle which hops from x to y. We now scale the function a as follows: for each $\varepsilon>0$, we denote

$$a_\varepsilon(x):=\varepsilon^{-d-2}a(x/\varepsilon),\quad x\in\mathbb{R}^d. \tag{4.2}$$

The Dirichlet form $(\mathcal{E}_{\mathrm{K}},D(\mathcal{E}_{\mathrm{K}}))$ which corresponds to the choice of function a as in (4.2) will be denoted by $(\mathcal{E}_\varepsilon,D(\mathcal{E}_\varepsilon))$.

Theorem 4.1. *Assume that the function a has compact support, and the value $a(x)$ only depends on $|x|$, i.e., $a(x)=\tilde a(|x|)$ for some function $\tilde a:[0,\infty)\to\mathbb{R}$. Further assume* *that the function $\varkappa(x,y)$ has the form $\varkappa(x-y)$ for some $\varkappa:\mathbb{R}^d\to\mathbb{C}$, and*

$$\lim_{y\to 0}\int_{\mathbb{R}^d}(\varkappa(x)-\varkappa(x+y))^2\,dx=0. \tag{4.3}$$

For each $l\in\mathbb{N}$, define a bilinear form $(\mathcal{E}_0,\mathcal{F}C_{\mathrm{b}}^\infty(C_0^\infty(\mathbb{R}^d),\Gamma))$ by

$$\mathcal{E}_0(F,G):=c\int_\Gamma \mu^{(l)}(d\gamma)\int_{\mathbb{R}^d}dx\,r(x,\gamma)\langle\nabla_x F(\gamma\cup x),\nabla_x G(\gamma\cup x)\rangle. \tag{4.4}$$

Here

$$c:=\frac{1}{2}\int_{\mathbb{R}^d}a(x)x_1^2\,dx$$

(x_1 denoting the first coordinate of $x\in\mathbb{R}^d$), ∇_x denotes the gradient in the x variable, and $\langle\cdot,\cdot\rangle$ stands for the scalar product in $\mathbb{R}^d$. Then, for any $F,G\in\mathcal{F}C_{\mathrm{b}}^\infty(C_0^\infty(\mathbb{R}^d),\Gamma)$,

$$\mathcal{E}_\varepsilon(F,G)\to\mathcal{E}_0(F,G)\quad \textit{as } \varepsilon\to 0.$$

Remark 4.1. Assume that the function $\varkappa$ is differentiable on $\mathbb{R}^d$. Denote

$$K(x,\delta):=\sup_{y\in B(x,\delta)}|\nabla\varkappa(y)|,\quad x\in\mathbb{R}^d,\quad \delta>0.$$

Here $B(x,\delta)$ denotes the closed ball in $\mathbb{R}^d$ centered at x and of radius δ. Assume that, for some $\delta>0$,

$$K(\cdot,\delta)\in L^2(\mathbb{R}^d,dx). \tag{4.5}$$

Then condition (4.3) is clearly satisfied. Note that condition (4.5) is slightly stronger than the condition $|\nabla\varkappa|\in L^2(\mathbb{R}^d,dx)$.

Proof of Theorem 4.1. Again we will only present the proof in the case $l=1$, omitting the upper index (1). We start with the following

Lemma 4.1. *Fix any $\Lambda\in\mathcal{B}_0(\mathbb{R}^d)$ and $\alpha\in(0,1]$. Then, under the conditions of Theorem 4.1,*

$$r(x+\varepsilon y,\gamma)^\alpha\to r(x,\gamma)^\alpha\quad \textit{in}\quad L^2(\Gamma\times\Lambda\times\mathbb{R}^d,\mu(d\gamma)\,dx\,dy\,a(y))\quad \textit{as}\quad \varepsilon\to 0.$$

Proof. We first prove the statement for $\alpha=1$. Thus, equivalently we have to prove that

$$r(x+\varepsilon y,\gamma)\to r(x,\gamma)\quad\text{in}\quad L^2(\Omega\times\Gamma\times\Lambda\times\mathbb{R}^d,\tilde{\mathbb{P}}(d\omega,d\gamma)\,dx\,dy\,a(y))\quad\text{as}\quad\varepsilon\to 0. \tag{4.6}$$

We have, using Jensen's inequality for conditional expectation,

$$\begin{aligned}
&\int_\Lambda dx\int_{\mathbb{R}^d}dy\,a(y)\int_{\Omega\times\Gamma}\tilde{\mathbb{P}}(d\omega,d\gamma)\,(r(x+\varepsilon y)-r(x,\gamma))^2\\
&\quad=\int_\Lambda dx\int_{\mathbb{R}^d}dy\,a(y)\int_{\Omega\times\Gamma}\tilde{\mathbb{P}}(d\omega,d\gamma)\tilde{\mathbb{E}}(Y(x+\varepsilon y)^2-Y(x)^2\mid\mathcal{F})^2\\
&\quad\le\int_\Lambda dx\int_{\mathbb{R}^d}dy\,a(y)\int_{\Omega\times\Gamma}\tilde{\mathbb{P}}(d\omega,d\gamma)(Y(x+\varepsilon y)^2-Y(x)^2)^2\\
&\quad=\int_\Lambda dx\int_{\mathbb{R}^d}dy\,a(y)\int_\Omega d\mathbb{P}\,(Y(x+\varepsilon y)^4+Y(x)^4-2Y(x+\varepsilon y)^2Y(x)^2).
\end{aligned} \tag{4.7}$$

Using the formula for moments of a Gaussian measure, we have

$$\begin{aligned}
&\int_\Omega Y(x+\varepsilon y)^4\,d\mathbb{P}\\
&= 3\Big(\int_{\mathbb{R}^d} \varkappa(x+\varepsilon y-u)^2\,du\Big)^2\\
&= 3\Big(\int_{\mathbb{R}^d} \varkappa(x-u)^2\,du\Big)^2\\
&= \int_\Omega Y(x)^4\,d\mathbb{P}.
\end{aligned} \tag{4.8}$$

Analogously, using condition (4.3) and the dominated convergence theorem, we get

$$\begin{aligned}
&\int_\Lambda dx \int_{\mathbb{R}^d} dy\, a(y) \int_\Omega d\mathbb{P}\, Y(x+\varepsilon y)^2 Y(x)^2\\
&\quad = \int_\Lambda dx \int_{\mathbb{R}^d} dy\, a(y) \Big[\int_{\mathbb{R}^d} \varkappa(x+\varepsilon y-u)^2\,du \cdot \int_{\mathbb{R}^d} \varkappa(x-u')^2\,du'\\
&\quad + 2\Big(\int_{\mathbb{R}^d} \varkappa(x+\varepsilon y-u)\varkappa(x-u)\,du\Big)^2\Big]\\
&\quad \to \int_\Lambda dx \int_{\mathbb{R}^d} dy\, a(y) \int_\Omega d\mathbb{P}\, Y(x)^4 \quad \text{as} \quad \varepsilon\to 0.
\end{aligned} \tag{4.9}$$

By (4.7)–(4.9), statement (4.6) follows.

To prove the result for $\alpha\in(0,1)$, it is now sufficient to show the following

Claim. Let $(\mathbf{A},\mathcal{A},m)$ be a measure space and let $m(A)<\infty$. Let $f_\varepsilon\in L^2(m)$, $f_\varepsilon\ge 0$, $\varepsilon\in[-1,1]$, and let $f_\varepsilon\to f_0$ in $L^2(m)$ as $\varepsilon\to 0$. Then, for each $\alpha\in(0,1)$, $f_\varepsilon^\alpha\to f_0^\alpha$ in $L^2(m)$ as $\varepsilon\to 0$.

By e.g. [1, Theorems 21.2 and 21.4], $f_\varepsilon\to f_0$ in $L^2(m)$ implies that

(i) $f_\varepsilon\to f_0$ in measure;

(ii) $\sup_{\varepsilon\in[-1,1]}\int f_\varepsilon^2\,dm<\infty$;

(iii) For each $\theta>0$ there exist $h\in L^1(m)$ and $\delta>0$ such that, for all $0<|\varepsilon|\le 1$ and for each $A\in\mathcal{A}$

$$\int_A h\,dm\le\delta \Rightarrow \int_A f_\varepsilon^2\,dm\le\theta.$$

Hence, for $\alpha\in(0,1)$, we get

a) $f_\varepsilon^\alpha\to f_0^\alpha$ in measure;

b) $\sup_{\varepsilon\in[-1,1]}\int f_\varepsilon^{2\alpha}\,dm\le \sup_{\varepsilon\in[-1,1]}\int(1+f_\varepsilon^2)\,dm<\infty$;

c) Let θ, h, and δ be as in (iii). Set $h':=h+\frac{\delta}{\theta}$. Clearly, $h\in L^1(m)$. Assume that, for some $A\in\mathcal{A}$, $\int_A h'\,dm\le\delta$. Hence $\int_A h\,dm\le\delta$, and therefore $\int_A f_\varepsilon^2\,dm\le\delta$ for all $0<|\varepsilon|\le 1$. Furthermore, we get $\int_A\frac{\delta}{\theta}\,dm\le\delta$, and therefore $m(A)\le\theta$. Now

$$\int_A f_\varepsilon^{2\alpha}\,dm\le\int_A(1+f_\varepsilon^2)\,dm\le 2\theta.$$

Applying again [1, Theorems 21.2 and 21.4], we conclude the claim. □

Fix any $F\in\mathcal{F}C_{\mathrm{b}}^\infty(C_0^\infty(\mathbb{R}^d),\Gamma)$. We have

$$\begin{aligned}
&\mathcal{E}_\varepsilon(F,F)\\
&= \frac12\int_\Gamma\mu(d\gamma)\int_{\mathbb{R}^d}dx\int_{\mathbb{R}^d}dy\,\varepsilon^{-d-2}a((x-y)/\varepsilon)r(x,\gamma)^{1/2}r(y,\gamma)^{1/2}(F(\gamma\cup x)-F(\gamma\cup y))^2
\end{aligned}$$

$$= \frac{1}{2}\int_\Gamma \mu(d\gamma)\int_{\mathbb{R}^d} dx \int_{\mathbb{R}^d} dy\, a(y) r(x+\varepsilon y,\gamma)^{1/2} r(x,\gamma)^{1/2}\left(\frac{F(\gamma\cup\{x+\varepsilon y\})-F(\gamma\cup x)}{\varepsilon}\right)^2.$$

Assume that $0<|\varepsilon|\le 1$. Noting that the function F is local (i.e., there exists $\Delta\in\mathcal{B}_0(\mathbb{R}^d)$ such that $F(\gamma)=F(\gamma_\Delta)$ for all $\gamma\in\Gamma$) and that the function a has a compact support, we conclude that there exists $\Lambda\in\mathcal{B}_0(\mathbb{R}^d)$ such that

$$\begin{aligned}\mathcal{E}_\varepsilon(F,F) = &\int_\Gamma \mu(d\gamma)\int_\Lambda dx\int_{\mathbb{R}^d} dy\, a(y) r(x+\varepsilon y,\gamma)^{1/2} r(x,\gamma)^{1/2}\\ &\times\left(\frac{F(\gamma\cup\{x+\varepsilon y\})-F(\gamma\cup x)}{\varepsilon}\right)^2.\end{aligned}\tag{4.10}$$

By the dominated convergence theorem

$$r(x,\gamma)^{1/2}\left(\frac{F(\gamma\cup\{x+\varepsilon y\})-F(\gamma\cup x)}{\varepsilon}\right)^2 \to r(x,\gamma)^{1/2}\langle\nabla_x F(\gamma\cup x),y\rangle^2\tag{4.11}$$

in $L^2(\Gamma\times\Lambda\times\mathbb{R}^d,\mu(d\gamma)\,dx\,dy\,a(y))$ as $\varepsilon\to 0$. By Lemma 4.1 with $\alpha=1/2$, (4.10) and (4.11)

$$\mathcal{E}_\varepsilon(F,F)\to\frac{1}{2}\int_\Gamma\mu(d\gamma)\int_\Lambda dx\int_{\mathbb{R}^d} dy\, a(y) r(x,\gamma)\langle\nabla_x F(\gamma\cup x),y\rangle^2.\tag{4.12}$$

Since $a(y)=\tilde a(|y|)$, for any $i,j\in\{1,\dots,d\}$, $i\ne j$, we have

$$\int_{\mathbb{R}^d} a(y) y_i y_j\, dy = 0$$

and

$$c=\frac{1}{2}\int_{\mathbb{R}^d} a(y) y_i^2\, dy,\quad i=1,\dots,d.$$

Therefore, by (4.12),

$$\mathcal{E}_\varepsilon(F,F)\to c\int_\Gamma\mu(d\gamma)\int_{\mathbb{R}^d} dx\, r(x,\gamma)|\nabla_x F(\gamma\cup x)|^2.$$

From here the theorem follows by the polarization identity. □

We will now show that the limiting form $(\mathcal{E}_0,\mathcal{F}C_{\mathrm{b}}^\infty(C_0^\infty(\mathbb{R}^d),\Gamma))$ is closable and its closure identifies a diffusion process.

In what follows, we will assume that the conditions of Theorem 4.1 are satisfied. We have

$$\begin{aligned}k(x,y)&=\int_{\mathbb{R}^d}\varkappa(x-u)\varkappa(y-u)\,du\\ &=\int_{\mathbb{R}^d}\varkappa(u-y)\varkappa(u-x)\,du=\int_{\mathbb{R}^d}\varkappa(u)\varkappa(u+y-x)\,du.\end{aligned}$$

Hence, by (4.3), the function $k(x,y)$ is continuous on $(\mathbb{R}^d)^2$. Thus, by Remark 3.1, $(Y(x))_{x\in X}$ is a Gaussian random field and formula (3.6) holds for all $(x,y)\in(\mathbb{R}^d)^2$.

Consider the semimetric

$$\begin{aligned}D(x,y):&=\frac{1}{2}\Big(\int_\Omega (Y(x)-Y(y))^2\,d\mathbb{P}\Big)^{1/2}\\ &=\frac{1}{2}\big(k(x,x)+k(y,y)-2k(x,y)\big)^{1/2}\\ &=\Big(\int_{\mathbb{R}^d}\varkappa(u)\big(\varkappa(u)-\varkappa(u+y-x)\big)\,du\Big)^{1/2},\quad x,y\in\mathbb{R}^d.\end{aligned}\tag{4.13}$$

The associated metric entropy $H(D,\delta)$ is defined as $H(D,\delta):=\log N(D,\delta)$, where $N(D,\delta)$ is the minimal number of points in a δ-net in $B(0,1)=\{x\in\mathbb{R}^d\mid |x|\le 1\}$

with respect to the semimetric D, i.e., points x_i such that the open balls centered at x_i and of radius δ (with respect to D) cover $B(0,1)$. The expression

$$J(D) := \int_0^1 \sqrt{H(D,\delta)}\, d\delta$$

is called the Dudley integral. The following result holds, see e.g. [4, Corollary 7.1.4] and the references therein.

Theorem 4.2. *Assume that $J(D) < \infty$. Then the Gaussian random field $(Y(x))_{x\in\mathbb{R}^d}$ has a continuous modification.*

Remark 4.2. Let $\varkappa$ be as in Remark 4.1. Then, by (4.13), for any $x, y \in B(0,1)$

$$\begin{aligned} D(x,y)^2 &\le \|\varkappa(\cdot)\|_{L^2(\mathbb{R}^d,dx)} \Big(\int_{\mathbb{R}^d} (\varkappa(u) - \varkappa(u+y-x))^2\, du\Big)^{1/2} \\ &\le \|\varkappa(\cdot)\|_{L^2(\mathbb{R}^d,dx)} \|K(\cdot,2)\|_{L^2(\mathbb{R}^d,dx)} |y-x|, \end{aligned}$$

where we assumed that $K(\cdot,2) \in L^2(\mathbb{R}^d, dx)$. Then $J(D) < \infty$, see e.g. [4, Example 7.1.5].

Denote by $\ddot{\Gamma}$ the space of all multiple configurations in $\mathbb{R}^d$. Thus, $\ddot{\Gamma}$ is the set of all Radon $\mathbb{Z}_+ \cup \{+\infty\}$-valued measures on $\mathbb{R}^d$, In particular, $\Gamma \subset \ddot{\Gamma}$. Analogously to the case of Γ, we define the vague topology on $\ddot{\Gamma}$ and the corresponding Borel σ-algebra $\mathcal{B}(\ddot{\Gamma})$.

Theorem 4.3. *Let $\varkappa(x, y)$ be of the form $\varkappa(x - y)$ for some $\varkappa \in L^2(\mathbb{R}^d, dx)$. Let $J(D) < \infty$. Let $l \in \mathbb{N}$ and $c > 0$. Then*

(i) The bilinear form $(\mathcal{E}_0, \mathcal{F}C_{\mathrm{b}}^\infty(C_0^\infty(\mathbb{R}^d), \Gamma))$ defined by (4.4) is closable on $L^2(\Gamma, \mu^{(l)})$ and its closure will be denoted by $(\mathcal{E}_0, D(\mathcal{E}_0))$.

(ii) There exists a conservative diffusion process

$$M^0 = \Big(\Omega^0, \mathcal{F}^0, (\mathcal{F}_t^0)_{t\ge0}, (\Theta_t^0)_{t\ge0}, (X^0(t))_{t\ge0}, (P_\gamma^0)_{\gamma\in\ddot{\Gamma}}\Big)$$

on $\ddot{\Gamma}$ which is properly associated with $(\mathcal{E}_0, D(\mathcal{E}_0))$. In particular, M^0 is $\mu^{(l)}$-symmetric and has $\mu^{(l)}$ as invariant measure. In the case $d \ge 2$, the set $\ddot{\Gamma} \setminus \Gamma$ is $\mathcal{E}^0$-exceptional, so that $\ddot{\Gamma}$ may be replaced by with Γ in the above statement.

Proof. We again discuss only the case $l = 1$, omitting the upper index (1). By (4.4), for any $F, G \in \mathcal{F}C_{\mathrm{b}}^\infty(C_0^\infty(\mathbb{R}^d), \Gamma)$,

$$\begin{aligned} \mathcal{E}_0(F,G) &= c\int_{\Omega\times\Gamma} \tilde{\mathbb{P}}(d\omega, d\gamma) \int_{\mathbb{R}^d} dx\, \tilde{\mathbb{E}}(Y(x,\omega)^2 \mid \mathcal{F}) \langle \nabla_x F(\gamma\cup x), \nabla_x G(\gamma \cup x)\rangle \\ &= \int_{\Omega\times\Gamma} \tilde{\mathbb{P}}(d\omega, d\gamma) \int_{\mathbb{R}^d} dx\, Y(x,\omega)^2 \\ &\quad\times \langle \nabla_x (F(\gamma\cup x) - F(\gamma)), \nabla_x (G(\gamma\cup x) - G(\gamma))\rangle. \end{aligned} \tag{4.14}$$

Fix $(\omega, \gamma) \in \Omega \times \Gamma$. Denote

$$f(x) := F(\gamma \cup x) - F(\gamma), \quad g(x) := G(\gamma\cup x) - G(\gamma).$$

Clearly, $f, g \in C_0^\infty(\mathbb{R}^d)$. In view of Theorem 4.2, $Y(x,\omega)^2$ is a continuous function of $x \in \mathbb{R}^d$. Hence, by [6, Theorem 6.2], the bilinear form

$$\mathcal{E}(f,g) := \int_{\mathbb{R}^d} \langle \nabla f(x), \nabla g(x)\rangle Y(x,\omega)^2 dx, \quad f, g \in C_0^\infty(\mathbb{R}^d),$$

is closable on $L^2(\mathbb{R}^d, |Y(x,\omega)|^2\, dx)$. Now the closability of $(\mathcal{E}_0, \mathcal{F}C_{\mathrm{b}}^\infty(C_0^\infty(\mathbb{R}^d), \Gamma))$ on $L^2(\Gamma, \mu^{(l)})$ follows by a straightforward generalization of the proof of [6, Theorem 6.3]. Part (ii) of the theorem can be shown completely analogously to [25, 29], see also [20]. $\square$

Remark 4.3. Heuristically, the generator of $(\mathcal{E}_0, D(\mathcal{E}_0))$ has the form

$$(LF)(\gamma) = \sum_{x\in\gamma} \Big(\Delta_x F(\gamma) + \Big\langle \frac{\nabla_x r(x, \gamma\setminus x)}{r(x, \gamma\setminus x)}\, , \nabla_x F(\gamma)\Big\rangle\Big).$$

Here, for $x \in \gamma$, $\nabla_x F(\gamma) := \nabla_y F(\gamma \setminus x \cup y)\big|_{y=x}$ and analogously Δ_x is defined. However, we should not expect that $r(x, \gamma)$ is differentiable in x.

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