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THE INTEGRATION BY PARTS FORMULA IN THE MEIXNER WHITE NOISE ANALYSIS

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Abstract. Using a general approach that covers the cases of Gaussian, Poissonian, Gamma, Pascal and Meixner measures on an infinite- dimensional space, we construct a general integration by parts formula for analysis connected with each of these measures. Our consideration is based on the constructions of the extended stochastic integral and the stochastic derivative that are connected with the structure of the extended Fock space.

Introduction

Let γ be the Gaussian measure on the Schwartz distributions space $D' = D'(\mathbb{R}_+)$, $L^2(D',\gamma)$ be the space of complex-valued functions on D', square integrable with respect to γ, $L^2(\mathbb{R}_+)$ be the space of complex-valued functions on $\mathbb{R}_+$ square integrable with respect to the Lebesgue measure. Denote by $\int_{\mathbb{R}_+} \circ(s)\,\widehat{d}W_s : L^2(D',\gamma)\otimes L^2(\mathbb{R}_+) \to L^2(D',\gamma)$ the extended (Hitsuda-Skorohod) stochastic integration operator (here $W_\cdot$ is a Wiener process). As is well known, for $G \in L^2(D',\gamma)$ and $h \in L^2(\mathbb{R}_+)$ such that $G\otimes h$ is integrable in the extended sense, $\int_{\mathbb{R}_+} Gh(s)\,\widehat{d}W_s \neq G\int_{\mathbb{R}_+} h(s)\,\widehat{d}W_s$, generally speaking (in spite of the fact that G does not depend on s). In fact, for stochastically differentiable $F \in L^2(D',\gamma)$,

$$\mathbf{E}\Big[F\int_{\mathbb{R}_+} Gh(s)\,\widehat{d}W_s\Big] = \mathbf{E}[G(\mathcal{D}F)(h)] \tag{0.1}$$

(here and below $(\mathcal{D}\circ)(h)$ is a stochastic derivative, $\mathbf{E}$ denotes an expectation), while if F and G are stochastically differentiable then

$$\mathbf{E}\Big[FG\int_{\mathbb{R}_+} h(s)\,\widehat{d}W_s\Big] = \mathbf{E}[G(\mathcal{D}F)(h) + F(\mathcal{D}G)(h)]. \tag{0.2}$$

Formula (0.2) is called *the integration by parts formula* in the Gaussian analysis.

If instead of the Gaussian case one considers the Poissonian one, formula (0.1) holds true with replacement of $\int_{\mathbb{R}_+} \circ(s)\,\widehat{d}W_s$ by the extended stochastic integration operator with respect to a Poissonian process and with a use of the corresponding stochastic derivative; but the analog of (0.2) is more complicated: instead of $\mathcal{D}$ in a one of terms in the right hand side one has to use a more complicated operator. A similar situation holds true if instead of $W_\cdot$, one considers a so-called *normal martingale with the chaos representation property (CRP)* (Wiener and Poissonian processes can be considered as particular cases of such martingales), see, e.g., [20] and references therein for details.

Another way of generalization of the Gaussian analysis consists in considering the so-called Meixner classes of probability measures and corresponding stochastic processes (these processes, with the exception of Wiener and Poissonian ones, have no CRP), see,

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e.g., [21, 9, 8, 3, 18, 19, 22, 15, 14]. During recent years such investigations became an object of interest of many specialists. In particular, in [2] a stochastic integral was introduced and studied for a wide class of stochastic processes and in [1] it was proved that a Meixner process satisfies the conditions [2]; in [9, 8] a stochastic integration theory with applications was constructed for Meixner processes and its generalizations; in [3] Meixner classes were derived as a renormalized square of white noise. In the papers [18, 19] E. W. Lytvynov offered a natural generalization of the classical results [21] to the infinite-dimensional case and made some applications in the stochastic analysis, his approach is based on the so-called Jacobi fields theory (e.g., [4]). In the paper [22] I. V. Rodionova constructed the infinite-dimensional "Meixner analysis" that is based on a generalization of results [19], considering the Gaussian, Poissonian, Gamma, Pascal and Meixner cases from a common point of view. It is worth noticing that the white noise in [22] is not a Lévy one, generally speaking (not time homogeneous). In the investigations of [18, 19, 22] an important role belongs to the so-called *extended Fock space* [17, 5], this space is naturally arises in the "Meixner analysis".

In the papers [16, 10, 12, 11] the author investigated the extended stochastic integral and stochastic derivatives in the so-called Gamma-analysis (i.e., in the analysis connected with the Gamma-measure – a particular case of the generalized Meixner measure [22]); the constructions in these papers are based on the structure of (the Gamma-version of) the extended Fock space. In the papers [15, 14] the author introduced and studied the extended stochastic integral and stochastic derivatives in the "Meixner analysis", these papers can be considered as enhanced generalizations of [16, 10, 12, 11]. The main aim of the present paper is to construct the integration by parts formula (the generalization of (0.2)) in the "Meixner analysis", using as a base the construction of the extended stochastic integral and the stochastic derivative from [15, 14].

The paper is organized in the following manner. In the first section we recall necessary definitions and results (the generalized Meixner measure, the corresponding orthogonal polynomials, the extended stochastic integral, the stochastic derivative etc.). For convenience of the reader, we give even a little more detailed presentation than it is necessary for understanding of the present paper. In the second section we construct the integration by parts formula.

1. Preliminaries

Let σ be a measure on $(\mathbb{R}_+, \mathcal{B}(\mathbb{R}_+))$ (here $\mathcal{B}$ denotes the Borel σ-algebra, $\mathbb{R}_+ := [0, \infty)$) satisfying the following assumptions:

1) σ is absolutely continuous with respect to the Lebesgue measure and the density is an infinite differentiable function on $\mathbb{R}_+$;
2) σ is a nondegenerate measure, i.e., for each nonempty open set $O \subset \mathbb{R}_+$, $\sigma(O) > 0$.

Remark 1.1. Note that these assumptions are the "simplest *sufficient* ones" for our considerations; actually it is possible to consider a much more general σ.

By D denote the set of all real-valued infinite differentiable functions on $\mathbb{R}_+$ with compact supports. This set can be naturally endowed with a (projective limit) topology of a nuclear space by analogy with, e.g., [6] (see, e.g., [15] for details), hence in what follows, we understand D as the corresponding *topological space*. Let D' be the set of linear continuous functionals on D, $\mathcal{H} := L^2(\mathbb{R}_+, \sigma)$ be the space of square integrable with respect to σ real-valued functions on $\mathbb{R}_+$. One can understand D' as the negative (endowed with the inductive limit topology) space of the chain

$$D' \supset \mathcal{H} \supset D$$

(e.g., [15]). Denote by $\langle\cdot,\cdot\rangle$ the dual pairing between elements of D' and D that is generated by the scalar product in $\mathcal{H}$; this notation will be preserved for tensor powers and complexifications of spaces.

Remark 1.2. Note that all scalar products and pairings in this paper are *real*. In this case norms are connected with scalar products by the formula $\|\cdot\|^2 := (\cdot,\bar{\cdot})$.

Let us fix arbitrary functions $\alpha,\beta:\mathbb{R}_+\to\mathbb{C}$ that are smooth and satisfy the following conditions:

$$\theta := -\alpha-\beta:\mathbb{R}_+\to\mathbb{R},\quad \eta:=\alpha\beta:\mathbb{R}_+\to\mathbb{R}_+,$$

θ and η are bounded on $\mathbb{R}_+$. Further, let for each $u\in\mathbb{R}_+$ $\widetilde{\upsilon}(\alpha(u),\beta(u),ds)$ be a probability measure on $\mathbb{R}$ that is defined by its Fourier transform,

$$\int_{\mathbb{R}} e^{i\lambda s}\widetilde{\upsilon}(\alpha(u),\beta(u),ds)=\exp\Big\{-i\lambda(\alpha(u)+\beta(u))$$
$$+2\sum_{m=1}^{\infty}\frac{(\alpha(u)\beta(u))^m}{m}\Big[\sum_{n=2}^{\infty}\frac{(-i\lambda)^n}{n!}(\beta^{n-2}(u)+\beta^{n-3}(u)\alpha(u)+\cdots+\alpha^{n-2}(u))\Big]^m\Big\},$$

$\upsilon(\alpha(u),\beta(u),ds):=\frac{1}{s^2}\widetilde{\upsilon}(\alpha(u),\beta(u),ds)$.

Let $\mathcal{C}(D')$ be the generated by cylinder sets σ-algebra on D'; denote by a subindex $\mathbb{C}$ the complexifications of spaces.

Definition 1.1. We say that a probability measure μ on the measurable space $(D',\mathcal{C}(D'))$ with a Fourier transform

$$\int_{D'} e^{i\langle x,\xi\rangle}\mu(dx)=\exp\Big\{\int_{\mathbb{R}_+}\sigma(du)\int_{\mathbb{R}}\upsilon(\alpha(u),\beta(u),ds)(e^{is\xi(u)}-1-is\xi(u))\Big\}$$

(here $\xi\in D$) is called *the generalized Meixner measure*.

Theorem 1.1. ([22]). *The generalized Meixner measure μ is a generalized stochastic process with independent values in the sense of* [7]*. The Laplace transform of μ is a holomorphic at $0\in D_{\mathbb{C}}$ function.*

Note that accordingly to the classical classification [21] (see also [18, 19, 22]) for *constants* α and β if $\alpha=\beta=0$ then μ is the Gaussian measure; if $\alpha\neq0$, $\beta=0$ then μ is the centered Poissonian measure; if $\alpha=\beta\neq0$ then μ is the centered Gamma measure; if $\alpha\neq\beta$, $\alpha\beta\neq0$, $\alpha,\beta:\mathbb{R}_+\to\mathbb{R}$ then μ is the centered Pascal measure; if $\alpha=\overline{\beta}$, $\mathrm{Im}(\alpha)\neq0$ then μ is the centered Meixner measure.

Denote by $(L^2):=L^2(D',\mu)$ the space of complex-valued square integrable with respect to μ functions on D'. Now we will construct a natural orthogonal basis in this space. Let $\widehat{\otimes}$ denote a symmetric tensor product. A function

$$D'\ni x\mapsto F(x)=\sum_{k=0}^{n}\langle x^{\otimes k},f^{(k)}\rangle\in\mathbb{C},\quad f^{(k)}\in D_{\mathbb{C}}^{\widehat{\otimes}k},\quad f^{(n)}\neq0$$

is called *a continuous polynomial on D' of power n*. Since the measure μ has a holomorphic at $0\in D_{\mathbb{C}}$ Laplace transform (Theorem 1.1), the set of all continuous polynomials on D' is dense in (L^2) ([23]). For $n\in\mathbb{Z}_+$ let $\mathcal{P}_n$ be the set of all continuous polynomials on D' of power $\leq n$, $\widetilde{\mathcal{P}}_n$ be the closure of $\mathcal{P}_n$ in (L^2). For $n\in\mathbb{N}$ denote $(L^2_n):=\widetilde{\mathcal{P}}_n\ominus\widetilde{\mathcal{P}}_{n-1}-$ the orthogonal difference in (L^2), $(L^2_0):=\mathbb{C}$. Then $(L^2)=\mathop{\oplus}\limits_{n=0}^{\infty}(L^2_n)$. For each $f^{(n)}\in D_{\mathbb{C}}^{\widehat{\otimes}n}$ we define $:\langle x^{\otimes n},f^{(n)}\rangle:$ as the orthogonal projection of $\langle x^{\otimes n},f^{(n)}\rangle$ onto (L^2_n). It follows from results of [22] that $:\langle x^{\otimes n},f^{(n)}\rangle:=\langle P_n(x),f^{(n)}\rangle$, where $P_n(x)\in {D'}^{\widehat{\otimes}n}$ and for

μ-almost all $x \in D'$ $P_0(x) = 1$, $P_1(x) = x$, and for all $f^{(n)} \in D_{\mathbb{C}}^{\widehat{\otimes} n}$, $h \in D_{\mathbb{C}}$

$$\begin{aligned}\langle P_{n+1}(x), f^{(n)}\widehat{\otimes} h\rangle &= \langle P_n(x), f^{(n)}\rangle\langle P_1(x), h\rangle \\ &\quad - n\langle P_n(x), Pr(\theta(\cdot)h(\cdot)f^{(n)}(\cdot, \cdot_2, \dots, \cdot_n))\rangle \\ &\quad - n\langle P_{n-1}(x), \langle f^{(n)}, h\rangle\rangle \\ &\quad - n(n-1)\langle P_{n-1}(x), Pr(\eta(\cdot)h(\cdot)f^{(n)}(\cdot, \cdot, \cdot_3 \dots, \cdot_n))\rangle,\end{aligned} \tag{1.1}$$

where Pr is the symmetrization operator,

$$\langle f^{(n)}, h\rangle := \int_{\mathbb{R}_+} f^{(n)}(s, \cdot_2, \dots, \cdot_n)h(s)\sigma(ds) \in D_{\mathbb{C}}^{\widehat{\otimes} n-1} \tag{1.2}$$

(moreover, it is proved in [22] that $\{\langle P_n(x), f^{(n)}\rangle : \ n \in \mathbb{Z}_+, \ f^{(n)} \in D_{\mathbb{C}}^{\widehat{\otimes} n}\}$ are Schefer polynomials with the generating function

$$\exp\Big\{ -\int_{\mathbb{R}_+} \Big(\frac{\lambda(s)^2}{2} + \sum_{n=3}^{\infty} \frac{\lambda(s)^n}{n}(\alpha(s)^{n-2} + \alpha(s)^{n-3}\beta(s) + \dots + \beta(s)^{n-2})\Big)\sigma(ds)$$
$$+ \Big\langle x, \lambda + \sum_{n=2}^{\infty} \frac{\lambda^n}{n}(\alpha^{n-1} + \alpha^{n-2}\beta + \dots + \beta^{n-1})\Big\rangle\Big\},$$

where $\lambda \in \mathcal{U}_0 \subset D_{\mathbb{C}}$, $x \in D'$, $\mathcal{U}_0$ is some neighborhood of $0 \in D_{\mathbb{C}}$).

Definition 1.2. We will call the polynomials $\langle P_n, f^{(n)}\rangle$, $f^{(n)} \in D_{\mathbb{C}}^{\widehat{\otimes} n}$, $n \in \mathbb{Z}_+$, *the generalized Meixner polynomials.*

Remark 1.3. Let α and β be constants. Then the generalized Meixner polynomials can be the generalized Hermite polynomials ($\alpha = \beta = 0$); the generalized Charlier polynomials ($\alpha \neq 0, \beta = 0$); the generalized Laguerre polynomials ($\alpha = \beta \neq 0$); the Meixner polynomials ($\alpha \neq \beta$, $\alpha\beta \neq 0$, $\alpha, \beta : \mathbb{R}_+ \to \mathbb{R}$); the Meixner-Pollaczek polynomials ($\alpha = \overline{\beta}$, $\mathrm{Im}(\alpha) \neq 0$).

Let $\mathcal{H}_{\mathrm{ext}}^{(n)}$ be a Hilbert space that is obtained as the closure of $D_{\mathbb{C}}^{\widehat{\otimes} n}$ ($n \in \mathbb{Z}_+$) with respect to the norm $|\cdot|_{\mathrm{ext}}$, this norm is generated by the scalar product

$$\langle f^{(n)}, g^{(n)}\rangle_{\mathrm{ext}} := \frac{1}{n!}\int_{D'} \langle P_n(x), f^{(n)}\rangle\langle P_n(x), g^{(n)}\rangle\mu(dx), \quad f^{(n)}, g^{(n)} \in D_{\mathbb{C}}^{\widehat{\otimes} n}.$$

It is not difficult to prove by analogy with [5] that the space $\mathcal{H}_{\mathrm{ext}}^{(n)}$ is, generally speaking, the orthogonal sum of $\mathcal{H}_{\mathbb{C}}^{\widehat{\otimes} n} \equiv L^2(\mathbb{R}_+, \sigma)_{\mathbb{C}}^{\widehat{\otimes} n}$ and some other Hilbert spaces (as a "limit case" one can consider $\eta = 0$, in this case $\mathcal{H}_{\mathrm{ext}}^{(n)} = \mathcal{H}_{\mathbb{C}}^{\widehat{\otimes} n}$). In this sense $\mathcal{H}_{\mathrm{ext}}^{(n)}$ *is an extension of* $\mathcal{H}_{\mathbb{C}}^{\widehat{\otimes} n}$.

One can give another explanation of the fact that $\mathcal{H}_{\mathrm{ext}}^{(n)}$ is a wider space than $\mathcal{H}_{\mathbb{C}}^{\widehat{\otimes} n}$. Namely, let $F^{(n)} \in \mathcal{H}_{\mathbb{C}}^{\widehat{\otimes} n}$ ($F^{(n)}$ is an equivalence class in $\mathcal{H}_{\mathbb{C}}^{\widehat{\otimes} n}$). We select a representative (a function) $\widetilde{F}^{(n)} \in F^{(n)}$ with a "zero diagonal", i.e., $\widetilde{F}^{(n)}(t_1, \dots, t_n) = 0$ if there exist $i, j \in \{1, \dots, n\}$ such that $i \neq j$ but $t_i = t_j$. This function generates an equivalence class $\widehat{F}^{(n)}$ in $\mathcal{H}_{\mathrm{ext}}^{(n)}$ that can be identified with $F^{(n)}$ (see [15] for details).

Definition 1.3. For $F^{(n)} \in \mathcal{H}_{\mathrm{ext}}^{(n)}$ ($n \in \mathbb{Z}_+$) we define $\langle P_n, F^{(n)}\rangle \in (L^2)$ as an (L^2)-limit

$$\langle P_n, F^{(n)}\rangle := \lim_{k\to\infty} \langle P_n, f_k^{(n)}\rangle,$$

where $(f_k^{(n)} \in D_{\mathbb{C}}^{\widehat{\otimes} n})_{k=1}^{\infty}$ is a sequence of "smooth" functions such that $f_k^{(n)} \to F^{(n)}$ in $\mathcal{H}_{\mathrm{ext}}^{(n)}$ as $k \to \infty$.

The following statement easily follows from the construction of the generalized Meixner polynomials (see also [22]).

Theorem 1.2. *A function $F \in (L^2)$ if and only if there exists a sequence of kernels $(F^{(n)} \in \mathcal{H}_{\mathrm{ext}}^{(n)})_{n=0}^{\infty}$ such that F can be presented in the form*

$$F = \sum_{n=0}^{\infty} \langle P_n, F^{(n)} \rangle, \tag{1.3}$$

where the series converges in (L^2), i.e., the (L^2)-norm of F

$$\|F\|_{(L^2)}^2 = \sum_{n=0}^{\infty} n! |F^{(n)}|_{\mathrm{ext}}^2 < \infty.$$

Furthermore, the system $\{\langle P_n, F^{(n)}\rangle,\ F^{(n)} \in \mathcal{H}_{\mathrm{ext}}^{(n)},\ n \in \mathbb{Z}_+\}$ plays a role of an orthogonal basis in (L^2) in the sense that for $F, G \in (L^2)$

$$(F,G)_{(L^2)} = \sum_{n=0}^{\infty} n! \langle F^{(n)}, G^{(n)} \rangle_{\mathrm{ext}},$$

where $F^{(n)}, G^{(n)}$ are the kernels from decompositions (1.3) for F, G. Finally, an explicit formula for the scalar product $\langle \cdot, \cdot \rangle_{\mathrm{ext}}$ has the form

$$\begin{aligned}
\langle F^{(n)}, G^{(n)} \rangle_{\mathrm{ext}} = & \sum_{\substack{k, l_j, s_j \in \mathbb{N}:\ j=1,\dots,k,\ l_1 > l_2 > \dots > l_k,\\ l_1 s_1 + \dots + l_k s_k = n}} \frac{n!}{l_1^{s_1} \dots l_k^{s_k} s_1! \dots s_k!} \\
& \times \int_{\mathbb{R}_+^{s_1+\dots+s_k}} F^{(n)}(\underbrace{t_1, \dots, t_1}_{l_1}, \dots, \underbrace{t_{s_1}, \dots, t_{s_1}}_{l_1}, \dots, \underbrace{t_{s_1+\dots+s_k}, \dots, t_{s_1+\dots+s_k}}_{l_k}) \\
& \times G^{(n)}(\underbrace{t_1, \dots, t_1}_{l_1}, \dots, \underbrace{t_{s_1}, \dots, t_{s_1}}_{l_1}, \dots, \underbrace{t_{s_1+\dots+s_k}, \dots, t_{s_1+\dots+s_k}}_{l_k}) \eta(t_1)^{l_1 - 1} \dots \eta(t_{s_1})^{l_1-1} \\
& \times \eta(t_{s_1+1})^{l_2-1} \dots \eta(t_{s_1+s_2})^{l_2-1} \dots \eta(t_{s_1+\dots+s_{k-1}+1})^{l_k-1} \dots \eta(t_{s_1+\dots+s_k})^{l_k-1} \\
& \times \sigma(dt_1) \dots \sigma(dt_{s_1+\dots+s_k}).
\end{aligned}$$

In particular, for $n=1$, $\langle F^{(1)}, G^{(1)} \rangle_{\mathrm{ext}} = \langle F^{(1)}, G^{(1)} \rangle = \int_{\mathbb{R}_+} F^{(1)}(s) G^{(1)}(s) \sigma(ds)$ (therefore $\mathcal{H}_{\mathrm{ext}}^{(1)} = \mathcal{H}_{\mathbb{C}}$); for $n=2$,

$$\langle F^{(2)}, G^{(2)} \rangle_{\mathrm{ext}} = \langle F^{(2)}, G^{(2)} \rangle + \int_{\mathbb{R}_+} F^{(2)}(s,s) G^{(2)}(s,s) \eta(s) \sigma(ds).$$

If $\eta = 0$ (this means that μ is the Gaussian or Poissonian measure) then $\langle F^{(n)}, G^{(n)} \rangle_{\mathrm{ext}} = \langle F^{(n)}, G^{(n)} \rangle$ for all $n \in \mathbb{Z}_+$; in general, $\langle F^{(n)}, G^{(n)} \rangle_{\mathrm{ext}} = \langle F^{(n)}, G^{(n)} \rangle + \dots$.

Now we recall the construction of the extended stochastic integral in the Meixner analysis (see [15] for a detailed presentation).

Let $F \in (L^2) \otimes \mathcal{H}_{\mathbb{C}}$. It follows from Theorem 1.2 that F can be presented in the form

$$F(\cdot) = \sum_{n=0}^{\infty} \langle P_n, F_{\cdot}^{(n)} \rangle, \quad F_{\cdot}^{(n)} \in \mathcal{H}_{\mathrm{ext}}^{(n)} \otimes \mathcal{H}_{\mathbb{C}} \tag{1.4}$$

with

$$\|F\|_{(L^2)\otimes\mathcal{H}_{\mathbb{C}}}^2 = \sum_{n=0}^{\infty} n! |F_{\cdot}^{(n)}|_{\mathcal{H}_{\mathrm{ext}}^{(n)} \otimes \mathcal{H}_{\mathbb{C}}}^2 < \infty.$$

Lemma 1.1. ([15]). *For given* $F^{(n)}_{\cdot} \in \mathcal{H}^{(n)}_{\mathrm{ext}} \otimes \mathcal{H}_{\mathbb{C}}$, $n \in \mathbb{Z}_+$, *we construct the element* $\widehat{F}^{(n)} \in \mathcal{H}^{(n+1)}_{\mathrm{ext}}$ *in the following way. Let* $\dot{F}^{(n)}_{\cdot} \in F^{(n)}_{\cdot}$ *be some representative (function) from the equivalence class* $F^{(n)}_{\cdot}$. *We set*

$$\tilde{\dot{F}}^{(n)}(u_1,\dots,u_n,u) := \begin{cases} \dot{F}^{(n)}_u(u_1,\dots,u_n), & \text{if} \quad u \neq u_1,\dots,u\neq u_n, \\ 0, & \text{in other cases,} \end{cases},$$

$\widehat{\dot{F}}^{(n)} := Pr\tilde{\dot{F}}^{(n)}$, *where* Pr *is the symmetrization operator. Let* $\widehat{F}^{(n)} \in \mathcal{H}^{(n+1)}_{\mathrm{ext}}$ *be the generated by* $\widehat{\dot{F}}^{(n)}$ *equivalence class in* $\mathcal{H}^{(n+1)}_{\mathrm{ext}}$. *This class is well-defined, does not depend on the representative* $\dot{F}^{(n)}_{\cdot}$, *and* $|\widehat{F}^{(n)}|_{\mathrm{ext}} \leq |F^{(n)}_{\cdot}|_{\mathcal{H}^{(n)}_{\mathrm{ext}}\otimes\mathcal{H}_{\mathbb{C}}}$.

Denote by $1_A(\cdot)$ the indicator of a set A. Let $\{M_s := \langle P_1, 1_{[0,s]}\rangle\}_{s\geq 0}$ be a Meixner random process (this process is a locally square integrable normal martingale with independent increments, see [15, 22] for more details).

Definition 1.4. Let $F \in (L^2)\otimes\mathcal{H}_{\mathbb{C}}$ be such that

$$\sum_{n=0}^{\infty}(n+1)!|\widehat{F}^{(n)}|^2_{\mathrm{ext}} < \infty, \tag{1.5}$$

where $\widehat{F}^{(n)} \in \mathcal{H}^{(n+1)}_{\mathrm{ext}}$ ($n \in \mathbb{Z}_+$) are constructed in Lemma 1.1 starting from the kernels $F^{(n)}_{\cdot} \in \mathcal{H}^{(n)}_{\mathrm{ext}} \otimes \mathcal{H}_{\mathbb{C}}$ from decomposition (1.4) for F. We define the extended stochastic integral with respect to the Meixner process $\int_{\mathbb{R}_+} F(s)\,\widehat{d}M_s \in (L^2)$ by setting

$$\int_{\mathbb{R}_+} F(s)\,\widehat{d}M_s := \sum_{n=0}^{\infty}\langle P_{n+1}, \widehat{F}^{(n)}\rangle.$$

Since $\|\int_{\mathbb{R}_+} F(s)\,\widehat{d}M_s\|^2_{(L^2)} = \sum_{n=0}^{\infty}(n+1)!|\widehat{F}^{(n)}|^2_{\mathrm{ext}} < \infty$, this definition is correct.

Remark 1.4. Note that for $h \in \mathcal{H}_{\mathbb{C}}$, $\int_{\mathbb{R}_+} h(s)\,\widehat{d}M_s = \langle P_1, h\rangle$.

It was proved in [15] that $\int_{\mathbb{R}_+} \circ(s)\,\widehat{d}M_s$ is a generalization of the Itô stochastic integral.

Finally we recall the construction of the stochastic derivative on (L^2) (see [14] for details).

Lemma 1.2. ([15]). *For a given* $F^{(n)} \in \mathcal{H}^{(n)}_{\mathrm{ext}}$ ($n \in \mathbb{N}$) *we construct an element* $F^{(n)}(\cdot) \in \mathcal{H}^{(n-1)}_{\mathrm{ext}} \otimes \mathcal{H}_{\mathbb{C}}$ *in the following way. Let* $\dot{F}^{(n)} \in F^{(n)}$ *be some representative (function) from the equivalence class* $F^{(n)}$. *We consider* $\dot{F}^{(n)}(\cdot)$ *(i.e., separate one argument of* $\dot{F}^{(n)}$*). Let* $F^{(n)}(\cdot) \in \mathcal{H}^{(n-1)}_{\mathrm{ext}} \otimes \mathcal{H}_{\mathbb{C}}$ *be the generated by* $\dot{F}^{(n)}(\cdot)$ *equivalence class in* $\mathcal{H}^{(n-1)}_{\mathrm{ext}} \otimes \mathcal{H}_{\mathbb{C}}$. *This class is well-defined, does not depend on the representative* $\dot{F}^{(n)}$, *and* $|F^{(n)}(\cdot)|_{\mathcal{H}^{(n-1)}_{\mathrm{ext}}\otimes\mathcal{H}_{\mathbb{C}}} \leq |F^{(n)}|_{\mathrm{ext}}$.

Using this result, for $F^{(n)} \in \mathcal{H}^{(n)}_{\mathrm{ext}}$ ($n \in \mathbb{N}$) and $h \in \mathcal{H}_{\mathbb{C}}$, we define $\langle F^{(n)}, h\rangle \in \mathcal{H}^{(n-1)}_{\mathrm{ext}}$ by formula (1.2), now ([15])

$$|\langle F^{(n)}, h\rangle|_{\mathrm{ext}} \leq |F^{(n)}|_{\mathrm{ext}}|h|_{\mathrm{ext}}. \tag{1.6}$$

Definition 1.5. For each $h \in \mathcal{H}_{\mathbb{C}}$ we define an operator $(\mathcal{D}\circ)(h) : (L^2) \to (L^2)$ with the domain

$$\mathrm{dom}(\mathcal{D}\circ)(h) = \{F \in (L^2) : \sum_{n=1}^{\infty} n!n|\langle F^{(n)}, h\rangle|^2_{\mathrm{ext}} < \infty\} \tag{1.7}$$

(here $F^{(n)} \in \mathcal{H}_{\mathrm{ext}}^{(n)}$, $n \in \mathbb{N}$ are the kernels from decomposition (1.3) for F) by the formula

$$(\mathcal{D}F)(h) := \sum_{n=1}^{\infty} n\langle P_{n-1}, \langle F^{(n)}, h\rangle\rangle. \tag{1.8}$$

The following statement is a reformulation of the corresponding results from [14].

Theorem 1.3. *The operators $(\mathcal{D}\circ)(h)$ $(h \in \mathcal{H}_{\mathbb{C}})$ and $\int_{\mathbb{R}_+} \circ h(s)\,\widehat{d}M_s$ are adjoint each other: for $F \in \mathrm{dom}(\mathcal{D}\circ)(h)$ (see (1.7)) and $G \in \mathrm{dom}\int_{\mathbb{R}_+} \circ h(s)\,\widehat{d}M_s \subset (L^2)$ (see (1.5))*

$$\mathbf{E}\Big[F\int_{\mathbb{R}_+} Gh(s)\,\widehat{d}M_s\Big] = \Big(\int_{\mathbb{R}_+} Gh(s)\,\widehat{d}M_s, F\Big)_{(L^2)} = \big(G, (\mathcal{D}F)(h)\big)_{(L^2)} = \mathbf{E}[G(\mathcal{D}F)(h)]$$

(cf. (0.1)), where $\mathbf{E}$ denotes an expectation. In particular, $(\mathcal{D}\circ)(h)$ is closed.

2. The integration by parts formula

As a base for construction of the integration by parts formula in the "Meixner analysis" we use relation (1.1). First we generalize this relation as follows. Let $h \in D_{\mathbb{C}}$, by Pr denote the symmetrization operator. By analogy with [22] we introduce, for each $n \in \mathbb{Z}_+$, creation and neutral operators $a_n^+(h) : \mathcal{H}_{\mathrm{ext}}^{(n)} \to \mathcal{H}_{\mathrm{ext}}^{(n+1)}$ and $a_n^0(h) : \mathcal{H}_{\mathrm{ext}}^{(n)} \to \mathcal{H}_{\mathrm{ext}}^{(n)}$ correspondingly by setting for $F^{(n)} \in D_{\mathbb{C}}^{\widehat{\otimes} n}$ $a_n^+(h)F^{(n)} := F^{(n)}\widehat{\otimes}h$, $a_n^0(h)F^{(n)} := nPr\big(\theta(\cdot)h(\cdot)F^{(n)}(\cdot, \cdot_2, \dots, \cdot_n)\big)$ and continue to $\mathcal{H}_{\mathrm{ext}}^{(n)}$ by continuity. This is possible because, as it follows from the calculations in [22], $|a_n^+(h)F^{(n)}|_{\mathrm{ext}} \le \sqrt{n+1}c_1(h)|F^{(n)}|_{\mathrm{ext}}$, $|a_n^0(h)F^{(n)}|_{\mathrm{ext}} \le nc_2(h)|F^{(n)}|_{\mathrm{ext}}$ with some positive $c_1(h), c_2(h)$. Further, let us define the annihilation operators $a_n^-(h) := a_n^+(h)^* : \mathcal{H}_{\mathrm{ext}}^{(n+1)} \to \mathcal{H}_{\mathrm{ext}}^{(n)}$: for all $F^{(n)} \in \mathcal{H}_{\mathrm{ext}}^{(n)}$, $G^{(n+1)} \in \mathcal{H}_{\mathrm{ext}}^{(n+1)}$ $\langle a_n^+(h)F^{(n)}, G^{(n+1)}\rangle_{\mathrm{ext}} = \langle F^{(n)}, a_n^-(h)G^{(n+1)}\rangle_{\mathrm{ext}}$. It is easy to calculate ([22]) that $a_n^-(h) = a_{n,1}^-(h) + a_{n,2}^-(h)$, where for $G^{(n+1)} \in D_{\mathbb{C}}^{\widehat{\otimes} n+1}$ $a_{n,1}^-(h)G^{(n+1)} = \langle G^{(n+1)}, h\rangle$ (see (1.2)), $a_{n,2}^-(h)G^{(n+1)} = nPr\big(\eta(\cdot)h(\cdot)G^{(n+1)}(\cdot, \cdot, \cdot_2, \dots, \cdot_n)\big)$ (it follows from (1.6) that $a_{n,1}^-(h)$ and, therefore, $a_{n,2}^-(h)$ can be continued to continuous operators acting from $\mathcal{H}_{\mathrm{ext}}^{(n+1)}$ to $\mathcal{H}_{\mathrm{ext}}^{(n)}$). Finally we note that $a_{n,1}^-(h)^* : \mathcal{H}_{\mathrm{ext}}^{(n)} \to \mathcal{H}_{\mathrm{ext}}^{(n+1)}$ has the form $a_{n,1}^-(h)^*F^{(n)} = F^{(n)} \diamond h$, $|F^{(n)} \diamond h|_{\mathrm{ext}} \le |F^{(n)}|_{\mathrm{ext}}|h|_{\mathrm{ext}}$, where ([14]) for $F^{(n)} \in \mathcal{H}_{\mathrm{ext}}^{(n)}$, $h \in D_{\mathbb{C}}$ $F^{(n)} \diamond h$ is an equivalence class in $\mathcal{H}_{\mathrm{ext}}^{(n+1)}$ that is generated by the function $\dot{F}^{(n)}(u_1, \dots, u_n)h(u)1_{\{u_1 \neq u, \dots, u_n \neq u\}}$, where $\dot{F}^{(n)} \in F^{(n)}$ is a representative of $F^{(n)}$; 1_A here and below denotes the indicator of an event A. Now (1.1) can be rewritten in the form

$$\begin{aligned} &\langle P_n, F^{(n)}\rangle\langle P_1, h\rangle = \langle P_{n+1}, a_n^+(h)F^{(n)}\rangle + \langle P_n, a_n^0(h)F^{(n)}\rangle \\ &+ n\langle P_{n-1}, a_{n-1,1}^-(h)F^{(n)}\rangle + n\langle P_{n-1}, a_{n-1,2}^-(h)F^{(n)}\rangle \\ &= \langle P_{n+1}, a_n^+(h)F^{(n)}\rangle + \langle P_n, a_n^0(h)F^{(n)}\rangle + n\langle P_{n-1}, a_{n-1}^-(h)F^{(n)}\rangle, \quad h \in D_{\mathbb{C}}, \end{aligned} \tag{2.1}$$

and this formula holds true for $F^{(n)} \in \mathcal{H}_{\mathrm{ext}}^{(n)}$.

Remark 2.1. One can interpret formula (2.1) as follows: *the operator of multiplication by the generalized Meixner process can be identified with the sum of the creation, neutral and annihilation operators.* The reader can find more details in [17, 18, 19, 22]; here we note only that such results are well-known in the Gaussian and Poissonian analysis (in the Gaussian case, in particular, the neutral operator is equal to zero).

Let now

$$F = \sum_{n=0}^{\infty}\langle P_n, F^{(n)}\rangle, \quad G = \sum_{m=0}^{\infty}\langle P_m, G^{(m)}\rangle \in (L^2), \tag{2.2}$$

$h \in D_{\mathbb{C}}$. As above, denote by $\mathbf{E}$ an expectation. Using (2.1) we obtain *informally*

$$GF \int_{\mathbb{R}_+} h(s)\, \widehat{d}M_s = GF\langle P_1, h\rangle = \sum_{n,m=0}^{\infty} \big[\langle P_m, G^{(m)}\rangle\langle P_{n+1}, a_n^+(h)F^{(n)}\rangle$$
$$+ \langle P_m, G^{(m)}\rangle\langle P_n, a_n^0(h)F^{(n)}\rangle + n\langle P_m, G^{(m)}\rangle\langle P_{n-1}, a_{n-1}^-(h)F^{(n)}\rangle\big],$$

therefore calculating (again *informally*!) the expectation of this product we have

$$\mathbf{E}\Big[GF \int_{\mathbb{R}_+} h(s)\, \widehat{d}M_s\Big] = \sum_{n=0}^{\infty} \big[(n+1)!\langle G^{(n+1)}, a_n^+(h)F^{(n)}\rangle_{\text{ext}}$$
$$+ n!\langle G^{(n)}, a_n^0(h)F^{(n)}\rangle_{\text{ext}} + 1_{\{n>0\}} n!\langle G^{(n-1)}, a_{n-1}^-(h)F^{(n)}\rangle_{\text{ext}}\big].$$

On the other hand, using (1.8) we obtain $(\mathcal{D}F)(h) = \sum_{n=1}^{\infty} n\langle P_{n-1}, a_{n-1,1}^-(h)F^{(n)}\rangle$, hence for $F \in \operatorname{dom}(\mathcal{D}\circ)(h)$, $G \in (L^2)$ $\mathbf{E}[G(\mathcal{D}F)(h)] = \sum_{n=1}^{\infty} n!\langle G^{(n-1)}, a_{n-1,1}^-(h)F^{(n)}\rangle_{\text{ext}}$ and (*informally*!)

$$\mathbf{E}\Big[FG \int_{\mathbb{R}_+} h(s)\, \widehat{d}M_s\Big] - \mathbf{E}[G(\mathcal{D}F)(h)] = \sum_{n=0}^{\infty} \big[(n+1)!\langle G^{(n+1)}, a_n^+(h)F^{(n)}\rangle_{\text{ext}}$$
$$+ n!\langle G^{(n)}, a_n^0(h)F^{(n)}\rangle_{\text{ext}} + 1_{\{n>0\}} n!\langle G^{(n-1)}, a_{n-1,2}^-(h)F^{(n)}\rangle_{\text{ext}}\big]$$
$$= \sum_{n=0}^{\infty} \big[(n+1)!\langle a_n^-(h)G^{(n+1)}, F^{(n)}\rangle_{\text{ext}} + n!\langle a_n^0(h)G^{(n)}, F^{(n)}\rangle_{\text{ext}}$$
$$+ 1_{\{n>0\}} n!\langle a_{n-1,2}^-(h)^* G^{(n-1)}, F^{(n)}\rangle_{\text{ext}}\big]$$
$$= \sum_{n=0}^{\infty} n!\langle F^{(n)}, (n+1)a_n^-(h)G^{(n+1)} + a_n^0(h)G^{(n)}$$
$$+ 1_{\{n>0\}}(a_{n-1}^+(h) - a_{n-1,1}^-(h)^*)G^{(n-1)}\rangle_{\text{ext}} = \mathbf{E}[F(\widetilde{\mathcal{D}}G)(h)],$$

where

$$(\widetilde{\mathcal{D}}G)(h) := \sum_{n=0}^{\infty}\langle P_n, (n+1)a_n^-(h)G^{(n+1)} + a_n^0(h)G^{(n)}$$
$$+ 1_{\{n>0\}}(a_{n-1}^+(h) - a_{n-1,1}^-(h)^*)G^{(n-1)}\rangle. \tag{2.3}$$

In order to make our calculations rigorous, let us estimate the (L^2)-norm of $(\widetilde{\mathcal{D}}G)(h)$. We have

$$\|(\widetilde{\mathcal{D}}G)(h)\|_{(L^2)}^2 = \sum_{n=0}^{\infty} n!|(n+1)a_n^-(h)G^{(n+1)} + a_n^0(h)G^{(n)}$$
$$+ 1_{\{n>0\}}(a_{n-1}^+(h) - a_{n-1,1}^-(h)^*)G^{(n-1)}|_{\text{ext}}^2$$
$$\le 4\sum_{n=0}^{\infty} n!\big[(n+1)^2|a_n^-(h)G^{(n+1)}|_{\text{ext}}^2 + |a_n^0(h)G^{(n)}|_{\text{ext}}^2$$
$$+ 1_{\{n>0\}}|a_{n-1}^+(h)G^{(n-1)}|_{\text{ext}}^2 + 1_{\{n>0\}}|a_{n-1,1}^-(h)^*G^{(n-1)}|_{\text{ext}}^2\big]$$
$$\le 4\sum_{n=0}^{\infty} n!\big[n^2c_1(h)^2 + n^2c_2(h)^2 + (n+1)^2c_1(h)^2 + (n+1)|h|_{\text{ext}}^2\big]|G^{(n)}|_{\text{ext}}^2 < \infty$$

if

$$\sum_{n=1}^{\infty} n!n^2|G^{(n)}|_{\text{ext}}^2 < \infty. \tag{2.4}$$

Therefore if $F \in \mathrm{dom}(\mathcal{D}\circ)(h)$ and $G \in (L^2)$ satisfies (2.4) then $G(\mathcal{D}F)(h)$ and $F(\widetilde{\mathcal{D}}G)(h)$ belong to $L^1(D', \mu)$. Moreover, it is easy to verify that under these conditions the expectation of $FG \int_{\mathbb{R}_+} h(s)\, \widehat{d}M_s$ is well-defined and we obtained the following.

Theorem 2.1. *Let $h \in D_{\mathbb{C}}$, $F \in \mathrm{dom}(\mathcal{D}\circ)(h)$, $G \in (L^2)$ satisfy* (2.4). *Then*

$$\mathbf{E}\Big[FG \int_{\mathbb{R}_+} h(s)\, \widehat{d}M_s\Big] = \mathbf{E}[G(\mathcal{D}F)(h) + F(\widetilde{\mathcal{D}}G)(h)], \tag{2.5}$$

where the operator $(\widetilde{\mathcal{D}}\circ)(h) : (L^2) \to (L^2)$ is defined by (2.3).

Formula (2.5) is called *the integration by parts formula* in the "Meixner analysis".

Note that estimate (2.4) (that describes the domain of $(\widetilde{\mathcal{D}}\circ)(h)$) is simple, universal for all subclasses of the generalized Meixner measure; but too restrictive. For example, in the Gaussian case ($\alpha = \beta = 0$, see Preliminaries) $(\widetilde{\mathcal{D}}\circ)(h) = (\mathcal{D}\circ)(h)$, but (2.4) is much more restrictive than the condition in (1.7). Fortunately, Theorem 2.1 can be enhanced in order to avoid this defect.

Lemma 2.1. *Let $h \in D_{\mathbb{C}}$. The operator $(\widetilde{\mathcal{D}}\circ)(h) : (L^2) \to (L^2)$ is closable.*

Proof. We have to show that if $\mathrm{dom}(\widetilde{\mathcal{D}}\circ)(h) \ni G_k \to 0$ in (L^2) as $k \to \infty$ and $(\widetilde{\mathcal{D}}G_k)(h) \to E$ in (L^2) as $k \to \infty$ then $E = 0$ in (L^2). Since for each k

$$G_k = \sum_{n=0}^{\infty} \langle P_n, G_k^{(n)} \rangle$$

and

$$\|G_k\|^2_{(L^2)} = \sum_{n=0}^{\infty} n! |G_k^{(n)}|^2_{\mathrm{ext}} \to 0$$

as $k \to \infty$, for each $n \in \mathbb{Z}_+$ $|G_k^{(n)}|_{\mathrm{ext}} \to 0$ as $k \to \infty$. Further,

$$\begin{aligned}(\widetilde{\mathcal{D}}G_k)(h) = \sum_{n=0}^{\infty} \langle P_n, E_k^{(n)} \rangle \equiv \sum_{n=0}^{\infty} \langle P_n, &(n+1)a_n^-(h)G_k^{(n+1)} + a_n^0(h)G_k^{(n)} \\ &+ 1_{\{n>0\}}(a_{n-1}^+(h) - a_{n-1,1}^-(h)^*)G_k^{(n-1)} \rangle,\end{aligned}$$

and therefore, for each $n \in \mathbb{Z}_+$, $|E_k^{(n)}|_{\mathrm{ext}} \to 0$ as $k \to \infty$ because the operators $a_n^-(h)$, $a_n^0(h)$, $a_{n-1}^+(h)$ and $a_{n-1,1}^-(h)^*$ are continuous. Finally, by the condition of the lemma

$$(\widetilde{\mathcal{D}}G_k)(h) \to E = \sum_{n=0}^{\infty} \langle P_n, E^{(n)} \rangle$$

in (L^2) as $k \to \infty$, therefore,

$$\|E - (\widetilde{\mathcal{D}}G_k)(h)\|^2_{(L^2)} = \sum_{n=0}^{\infty} n! |E^{(n)} - E_k^{(n)}|^2_{\mathrm{ext}} \to 0$$

as $k \to \infty$, hence, for each $n \in \mathbb{Z}_+$, $|E^{(n)} - E_k^{(n)}|_{\mathrm{ext}} \to 0$ as $k \to \infty$. But in this case, for each $n \in \mathbb{Z}_+$, $|E^{(n)}|_{\mathrm{ext}} = 0$ (because

$$|E^{(n)}|_{\mathrm{ext}} = |E^{(n)} - E_k^{(n)} + E_k^{(n)}|_{\mathrm{ext}} \le |E^{(n)} - E_k^{(n)}|_{\mathrm{ext}} + |E_k^{(n)}|_{\mathrm{ext}} \underset{k\to\infty}{\to} 0),$$

hence $E = 0$ in (L^2). □

Denote by $(\widehat{\mathcal{D}}\circ)(h)$ the closure of $(\widetilde{\mathcal{D}}\circ)(h)$. Now we have the following "enhanced" variant of Theorem 2.1.

Theorem 2.2. *Let $h \in D_{\mathbb{C}}$, $F \in \operatorname{dom}(\mathcal{D}\circ)(h)$, $G \in \operatorname{dom}(\widehat{\mathcal{D}}\circ)(h)$, and the expectation of $FG \int_{\mathbb{R}_+} h(s)\, \widehat{d}M_s$ be well-defined. Then*

$$\mathbf{E}\Big[FG \int_{\mathbb{R}_+} h(s)\, \widehat{d}M_s\Big] = \mathbf{E}[G(\mathcal{D}F)(h) + F(\widehat{\mathcal{D}}G)(h)].$$

Remark 2.2. The expectation of $FG \int_{\mathbb{R}_+} h(s)\widehat{d}M_s$ is well-defined if, for example, kernels from decompositions (2.2) for F and G satisfy the estimates

$$\sum_{n=0}^{\infty} n!n|F^{(n)}|^2_{\mathrm{ext}} < \infty, \quad \sum_{n=0}^{\infty} n!n|G^{(n)}|^2_{\mathrm{ext}} < \infty.$$

Remark 2.3. As we noted above, in the Gaussian case $(\widetilde{\mathcal{D}}G)(h) = (\mathcal{D}G)(h)$, and therefore in the integration by parts formula one can use "nonsmooth" $h \in \mathcal{H}_{\mathbb{C}}$. In the general case such a generalization is impossible; but one can easily generalize the results of Theorems 2.1,2.2 at least for *bounded σ-a.e.* $h \in \mathcal{H}_{\mathbb{C}}$.

Remark 2.4. In the paper [13] the stochastic integral and the stochastic derivative $(\mathcal{D}\circ)(h)$ on the so-called *parametrized Kondratiev-type spaces* of test and generalized functions were considered ((L^2) is a particular case of these spaces). Note that the integration by parts formula (2.5) holds true for a test function F and a generalized function G. As for the case of generalized functions F and G, in this case, expectations are not determined, but can be replaced by the corresponding pairings.

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