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ANOTHER FORM OF SEPARATION AXIOMS

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Abstract. It is the object of this paper to introduce the $(1,2)^*$pre-D_k axioms for $k = 0, 1, 2$.

1. Introduction

In 1982, Mashhour et al [9] introduced the notion of pre-open sets and the pre-closed sets were defined in [4]. Ashish Kar and Bhattacharya [2], in 1990, continued their work on pre-open sets and offered another set of separation axioms analogous to the semi separation axioms defined by Maheshwari and Prasad [8]. Caldas [10] defined a new class of sets called semi-Difference (briefly sD) sets by using the semi-open sets [5], and introduced the semi-D_i spaces for $i = 0, 1, 2$.

The purpose of this paper is to introduce some pre-separation axioms in bitopological spaces. To define and investigate the axioms we use the $(1,2)^*$pre-open sets [6] introduced by Lellis Thivagar and Ravi. We call these axioms as $(1,2)^*$pre-T_0, $(1,2)^*$pre-T_1 and $(1,2)^*$pre-T_2. We also define $(1,2)^*$pre-Difference sets and utilize them to define the $(1,2)^*$pre-D_i, $i = 0, 1, 2$, axioms. We prove a bitopological space is $(1,2)^*$pre-D_1 if and only if it is $(1,2)^*$pre-D_2.

We recall some definitions and concepts which are useful in the following sections.

2. Preliminaries

In this section unless it is explicitly stated X is a topological space (X, τ). For a $A \subset X$, the interior and closure of A in X are denoted by $int(A)$ and $cl(A)$ respectively.

Definition 2.1. A space X is called

(i). Pre-T_0 [2]] iff to each pair of distinct points x, y in X, there exists a pre-open set containing one of the points but not the other.

(ii). Pre-T_1 [2] iff to each pair of distinct points x, y of X, there exists a pair of pre-open sets one containing x but not y and other containing y but not x.

(iii). Pre-T_2 [2] iff to each pair of distinct points x, y of X, there exists a pair of disjoint pre-open sets one containing x and the other containing y.

Definition 2.2. A subset A of X is called a semi-Difference set (in short sD-set) if there are two semi-open sets O_1, O_2 in X such that $O_1 \neq X$ and $A = O_1 \setminus O_2$.

Definition 2.3. A space X is called

(i). Semi-D_0 if for $x, y \in X$, $x \neq y$, there exists a sD-set of X containing one of x and y but not the other.

(ii). Semi-D_1 if for $x, y \in X$, $x \neq y$, there exists a pair of sD-sets one containing x but not y and the other containing y but not x.

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(iii). Semi-D_2 if for $x,\, y \in X$, $x \neq y$, there exist disjoint sD-sets S_1 and S_2 such that $x \in S_1$ and $y \in S_2$.

A subset A of a bitopological space $(X,\, \tau_1,\, \tau_2)$ is called $\tau_{1,2}$-open [6] if $A = S_1 \cup S_2$ where $S_1 \in \tau_1$ and $S_2 \in \tau_2$, and $\tau_{1,2}$-closed if A^c is $\tau_{1,2}$-open in X. We write $\tau_1\tau_2$-interior of A and $\tau_1\tau_2$-closure of A, in short form, $\tau_1\tau_2$-$int(A)$ and $\tau_1\tau_2$-$cl(A)$ respectively. $\tau_1\tau_2$-$int(A)$ is the union of all $\tau_{1,2}$-open sets contained in A and $\tau_1\tau_2$-$cl(A)$ is the intersection of all $\tau_{1,2}$-closed sets containing A.

Definition 2.4. A subset A of X is called $(1,2)^*$pre-open [6] if $A \subset \tau_1\tau_2\text{-}int(\tau_1\tau_2\text{-}cl(A))$ and $(1,2)^*$pre-closed if its complement in X is $(1,2)^*$pre-open. Or equivalently, $\tau_1\tau_2\text{-}cl(\tau_1\tau_2\text{-}int(A)) \subset A$.

The family of all $(1,2)^*$pre-open sets of X is denoted by $(1,2)^*PO(X)$. $(1,2)^*$pre-closure of A denoted by $(1,2)^*pcl(A)$ is the intersection of all $(1,2)^*$pre-closed sets containing A.

A subset A of X is $(1,2)^*$pre-closed if and only if $(1,2)^*pcl(A) = A$. Note that every $\tau_{1,2}$-open set is $(1,2)^*$pre-open.

Definition 2.5. A map f **:** $(X,\, \tau_1,\, \tau_2) \rightarrow (Y,\, \sigma_1,\, \sigma_2)$is called

(i). $(1,2)^*$pre-irresolute if the inverse image of every $(1,2)^*$pre-open set in Y is $(1,2)^*$pre-open in X.

(ii). Strongly $(1,2)^*$pre-open [6] if the image of every $(1,2)^*$pre-open if the image of every $(1,2)^*$pre-open set in X is $(1,2)^*$pre-open in Y.

In the next section we introduce the $(1,2)^*$pre-T_k-spaces for $k = 0,\, 1,\, 2$. In the following sections by X and Y we mean bitopological spaces $(X,\, \tau_1,\, \tau_2)$ and $(Y,\, \sigma_1,\, \sigma_2)$ respectively.

3. $(1,2)^*$PRE-T_k-SPACES

Definition 3.1. A bitopological space X is said to be $(1,2)^*$pre-T_0 iff for $x,\, y \in X$, $x \neq y$, there exists an $(1,2)^*$pre-open set containing only one of x and y but not the other.

Now we proceed to prove that every bitopological space is $(1,2)^*$pre-T_0.

Lemma 3.2. *If for some $x \in X$, $\{x\}$ is $(1,2)^*$pre-open then $x \notin (1,2)^*pcl(\{y\})$ for all $y \neq x$.*

Proof. If $\{x\}$ is $(1,2)^*$pre-open for some $x \in X$, then $X \setminus \{x\}$ is $(1,2)^*$pre-closed and $x \notin X \setminus \{x\}$. If $x \in (1,2)^*pcl(\{y\})$ for some $y \neq x$, then x, y both are in all the $(1,2)^*$pre-closed sets containing y which implies that $x \in X \setminus \{x\}$ which is not true. Therefore, $x \notin (1,2)^*pcl(\{y\})$. □

Theorem 3.3. *In a space X, distinct points have distinct $(1,2)^*$pre-closures.*

Proof. Let $x,\, y \in X$, $x \neq y$. Take $A = \{x\}^c$. Then $\tau_1\tau_2\text{-}cl(A) = A$ or X.

Case (a). If $\tau_1\tau_2\text{-}cl(A) = A$, then A is $\tau_{1,2}$-closed and hence $(1,2)^*$pre-closed. Then $X \setminus A = \{x\}$ is $(1,2)^*$pre-open, not containing y. Therefore, by Lemma 3.2, $x \notin (1,2)^*pcl(\{y\})$ and $y \in (1,2)^*pcl(\{y\})$ which implies that

$$(1,2)^*pcl(\{x\}) \qquad \text{and} \qquad (1,2)^*pcl(\{y\})$$

are distinct.

Case (b). If $\tau_1\tau_2\text{-}cl(A) = X$, then A is $(1,2)^*$pre-open and hence $\{x\}$ is $(1,2)^*$pre-closed which shows that $(1,2)^*pcl(\{x\}) = \{x\}$ which is not equal to $(1,2)^*pcl(\{y\})$. □

Theorem 3.4. *In a space X, if distinct points have distinct $(1,2)^*$pre-closures then X is $(1,2)^*$pre-T_0.*

Proof. Let x, $y \in X$, $x \neq y$. Then $(1,2)^*pcl(\{x\})$ is not equal to $(1,2)^*pcl(\{y\})$. Then there exists $z \in X$ such that $z \in (1,2)^*pcl(\{x\})$ but $z \notin (1,2)^*pcl(\{y\})$ or $z \in (1,2)^*pcl(\{y\})$ but $z \notin (1,2)^*pcl(\{x\})$. Without loss of generality, let $z \in (1,2)^*pcl(\{x\})$ but $z \notin (1,2)^*pcl(\{y\})$. If $x \in (1,2)^*pcl(\{y\})$, then $(1,2)^*pcl(\{x\})$ is contained in $(1,2)^*pcl(\{y\})$ and therefore, $z \in (1,2)^*pcl(\{y\})$, which is a contradiction. Thus we get $x \notin (1,2)^*pcl(\{y\})$. This implies that $x \in (1,2)^*pcl((\{y\})^c)$. Therefore, X is $(1,2)^*$pre-T_0. □

Theorem 3.5. *Every bitopological space is $(1,2)^*$pre-T_0.*

Proof. Follows from Theorem 3.3 and Theorem 3.4. □

Remark 3.6. It is observed that every T_0-space is pre-T_0 but not the converse [2]. Here we note that if a space X is T_0 with respect to τ_1 or τ_2 then X is $(1,2)^*$pre-T_0. But if X is $(1,2)^*$pre-T_0, it is not necessary that (X, τ_1) is T_0 or (X, τ_2) is T_0, as shown in the following example.

Example 3.7. Let $X = \{a, b, c\}$. $\tau_1 = \{\emptyset, \{a,b\}, X\}$, $\tau_2 = \{\emptyset, \{b,c\}, X\}$. Then X is $(1,2)^*$pre-T_0 but both *(X, τ_1)* and *(X, τ_2)* are T_0.

Definition 3.8. A space X is called $(1,2)^*$pre-T_1 iff for x, $y \in X$, $x \neq y$, there exist U, $V \in (1,2)^*PO(X)$ such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$.

Remark 3.9. It is obvious that every $(1,2)^*$pre- T_1 space is $(1,2)^*$pre-T_0 but the converse is not true in general as illustrated in the next example.

Example 3.10. Let $X = \{a, b, c\}$. $\tau_1 = \{\emptyset, \{a\}, X\}$, $\tau_2 = \{\emptyset, \{b\}, X\}$. Then X is $(1,2)^*$pre-T_0 but not $(1,2)^*$pre-T_1.

Theorem 3.11. *In a space X, the following statements are equivalent.*

(i). *X is $(1,2)^*$pre-T_1.*
(ii). *For each $x \in X$, $\{x\}$ is $(1,2)^*$pre-closed in X.*
(iii). *Each subset of X is the intersection of all $(1,2)^*$pre-open sets containing it.*
(iv). *The intersection of all $(1,2)^*$pre-open sets containing the point $x \in X$ is $\{x\}$.*

Proof. $(i) \Rightarrow (ii)$.

Let $x \in X$. If $y \in X$ and $x \neq y$ then there exists an $(1,2)^*$pre-open set U_y such that $y \in U_y$. Hence $y \in U_y \subset \{x\}^c$. Therefore, $\{x\}^c = \bigcup \{U_y : y \in \{x\}^c\}$ which is $(1,2)^*$pre-open and so $\{x\}$ is $(1,2)^*$pre-closed in X.

$(ii) \Rightarrow (iii)$.

Let $A \subset X$ and $y \notin A$. Then $A \subset \{y\}^c$ and $\{y\}^c$ is $(1,2)^*$pre-open in X and $A = \bigcap \{\{y\}^c : y \in A^c\}$ which is the intersection of all $(1,2)^*$pre-open sets containing A.

$(iii) \Rightarrow (iv)$.

Obvious.

$(iv) \Rightarrow (i)$.

Let x, $y \in X$, $x \neq y$. By our assumption, there exist atleast an $(1,2)^*$pre-open set containing x but not y and also an $(1,2)^*$pre-open set containing y but not x. Therefore, X is $(1,2)^*$pre-T_1. □

Definition 3.12. A space X is called $(1,2)^*$pre-T_2 iff for x, $y \in X$, $x \neq y$, there exist disjoint $(1,2)^*$pre-open sets U, V in X such that $x \in U$ and $y \in V$.

Remark 3.13. $(1,2)^*$pre-T_2ness implies $(1,2)^*$pre-T_1ness but the converse is not true in general. In Example 3.7, X is $(1,2)^*$pre-T_1 but not $(1,2)^*$pre-T_2.

Definition 3.14. A subset O of X is said to be $(1,2)^*$pre-neighbourhood of a point $x \in X$ iff there exists an $(1,2)^*$pre-open set U such that $x \in U \subset O$.

Theorem 3.15. *For a space X the following statements are equivalent.*

(i). *X is $(1,2)^*$pre-T_2*

(ii). *If $x \in X$, then for each $y \neq x$, there is an $(1,2)^*$pre-neighbourhood $N(x)$ of x such that $y \notin (1,2)^*pcl(N(x))$.*

(iii). *For each $x \in \{(1,2)^*pcl(N)$: N is an $(1,2)^*$pre-neighbourhood of $x\} = \{x\}$.*

Proof. $(i) \Rightarrow (ii)$.

Let $x \in X$. If $y \in X$ is such that $y \neq x$, there exist disjoint $(1,2)^*$pre-open sets U, V such that $x \in U$ and $y \in V$. Then $x \in U \subset X \setminus V$ which implies that $X \setminus V$ is an $(1,2)^*$pre-neighbourhood of x. Also $X \setminus V$ is $(1,2)^*$pre-closed and $y \notin X \setminus V$. Let $N(x) = X \setminus V$. Then $y \notin (1,2)^*pcl(N(x))$.

$(ii) \Rightarrow (iii)$.

Obvious.

$(iii) \Rightarrow (i)$.

Let $x, y \in X$, $x \neq y$. By hypothesis, there is atleast an $(1,2)^*$pre-neighbourhood N of x such that $y \notin (1,2)^*pcl(N)$. We have $x \notin X \setminus (1,2)^*pcl(N)$ is $(1,2)^*$pre-open. Since N is an $(1,2)^*$pre-neighbourhood of x, there exists $U \in (1,2)^*PO(X)$ such that $x \in U \subset N$ and $U \cap (X \setminus (1,2)^*pcl(N)) = \emptyset$. Hence X is $(1,2)^*$pre-T_2. □

Definition 3.16. A space X is said to be $(1,2)^*$pre-regular if for each $(1,2)^*$pre-closed set F and each point $x \notin F$ there exist disjoint $(1,2)^*$pre-open sets U and V such that $x \in U$ and $F \subset V$.

Theorem 3.17. *An $(1,2)^*$pre-T_0 space is $(1,2)^*$pre-T_2 if it is $(1,2)^*$pre-regular.*

Proof. Let X be $(1,2)^*$pre-T_0 and $(1,2)^*$pre-regular. If $x, y \in X$, $x \neq y$, there exists $U \in (1,2)^*PO(X)$ such that U contains one of x and y, say x but not y. Then $X \setminus U$ is $(1,2)^*$pre-closed and $x \notin X \setminus U$. Since X is $(1,2)^*$pre-regular, there exist disjoint $(1,2)^*$pre-open sets V_1 and V_2 such that $x \in V_1$ and $X \setminus U \subset V_2$. Thus $x \in V_1$ and $y \in V_2$, $V_1 \cap V_2 = \emptyset$. Hence X is $(1,2)^*$pre-T_2. □

Theorem 3.18. *If $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an injective, $(1,2)^*$pre-irresolute map and Y is $(1,2)^*$pre-T_2 then X is $(1,2)^*$pre-T_2.*

Proof. Let $x, y \in X$, $x \neq y$. Since f is injective, $f(x) \neq f(y)$ in Y and there exist disjoint $(1,2)^*$pre-open sets U, V such that $f(x) \in U$ and $f(y) \in V$. Let $G = f^{-1}(U)$ and $H = f^{-1}(V)$. Then $x \in G$, $y \in H$ and $G, H \in (1,2)^*PO(X)$. Also $G \cap H = f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = \emptyset$. Thus X is $(1,2)^*$pre-T_2. □

4. Pre-difference axioms

Definition 4.1. A subset A of X is called $(1,2)^*$pre-difference set (briefly $(1,2)^*pD$-set) if there are two $(1,2)^*$pre-open sets P_1 and P_2 in X, $P_1 \neq X$ such that $A = P_1 \setminus P_2$.

Remark 4.2. It is evident that each $(1,2)^*$pre-open set is an $(1,2)^*pD$-set.

Now we define another set of separation axioms called $(1,2)^*$pre-D_i, $i = 0, 1, 2$ by using the $(1,2)^*pD$-sets.

Definition 4.3. A space X is said to be

(i). $(1,2)^*$pre-D_0 if for $x, y \in X$, $x \neq y$, there exists an $(1,2)^*pD$-set containing one of x and y but not the other.

(ii). $(1,2)^*$pre-D_1 if for $x, y \in X$, $x \neq y$, there exist $(1,2)^*pD$-sets U, V in X such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$.

(iii). $(1,2)^*$pre-D_2 if for $x,\ y \in X,\ x \neq y$, there exist disjoint $(1,2)^*pD$-sets U, V in X such that $x \in U$ and $y \in V$.

Remark 4.4. (i). Every $(1,2)^*$pre-T_i space is $(1,2)^*$pre-D_i, $i = 0,\ 1,\ 2$ respectively.
(ii). If X is $(1,2)^*$pre-D_i then it is $(1,2)^*$pre-D_{i-1}, $i = 1,\ 2$.

Remark 4.5. $(1,2)^*$pre-D_i ness does not imply $(1,2)^*$pre-T_i ness for $i = 1,\ 2$ respectively. In example 3.10, X is $(1,2)^*$pre-D_1 but not $(1,2)^*$pre-T_1 and in Example 3.7, X is not $(1,2)^*$pre-T_2 but $(1,2)^*$pre-D_2.

The next example shows that $(1,2)^*$pre-D_0 does not imply $(1,2)^*$pre-D_1.

Example 4.6. Let $X = \{a,\ b\}$, $\tau_1 = \{\emptyset,\ X\}$, $\tau_2 = \{\emptyset,\ \{a\},\ X\}$. Then X is $(1,2)^*$pre-D_0 but not $(1,2)^*$pre-D_1.

Theorem 4.7. *A space X is $(1,2)^*$pre-D_0 if and only if it is $(1,2)^*$pre-T_0.*

Proof. Suppose that X is $(1,2)^*$pre-D_0. Let $x,\ y \in X,\ x \neq y$. Then there exists an $(1,2)^*pD$-set U such that U contains x but not y, say. As U is an $(1,2)^*pD$-set, it is possible to write $U = P_1 \setminus P_2$ where $P_1 \neq X$ and $P_1,\ P_2 \in (1,2)PO(X)$. Now there arises two cases. (i)$y \notin P_1$ (ii) $y \in P_1$ and $y \in P_2$.

Case (i). $y \notin P_1$ and $x \in P_1 \setminus P_2$ implies that $x \in P_1$ and $y \notin P_1$.

Case (ii). $y \in P_1$ and $y \in P_2$. $x \in P_1 \setminus P_2$ implies that $x \notin P_2$. Thus $y \in P_2$ and $x \notin P_2$.

Thus in both the cases, we obtain that X is $(1,2)^*$pre-T_0. Conversely, if X is $(1,2)^*$pre-T_0, by Remark 4.4, X is $(1,2)^*$pre-D_0. □

It has been showed in section 3, that an $(1,2)^*$pre-T_2-space is $(1,2)^*$pre-T_1 but not the converse. But in the case of pre-Difference axioms, we prove that an $(1,2)^*$pre-D_1-space is $(1,2)^*$pre-D_2 and so the $(1,2)^*$pre-D_1-space coincides with $(1,2)^*$pre-D_2-space.

Theorem 4.8. *A space X is $(1,2)^*$pre-D_1 if and only if X is $(1,2)^*$pre-D_2.*

Proof. **Necessity.** Let $x,\ y \in X,\ x \neq y$. Then there exist $(1,2)^*pD$-sets U, V in X such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$. Let $U = P_1 \setminus P_2$ and $V = P_3 \setminus P_4$ where $P_i \in (1,2)^*PO(X)$, $i = 1,\ 2,\ 3,\ 4$ and $P_1 \neq X$, $P_3 \neq X$. It is evident that $x \notin V$ implies the two possibilities, (i) $x \in P_3 \cap P_4$ (ii) $x \notin P_3$.

Case (i). $x \in P_3 \cap P_4$. We have $x \in P_4$ and $y \in P_3 \setminus P_4$ and $P_4 \cap (P_3 \setminus P_4) = \emptyset$ are disjoint.

Case (ii). $x \notin P_3$. $y \notin U$ implies that either $y \in P_1$ and $y \in P_2$ or $y \notin P_1$.

Sub Case (a). $y \in P_1$ and $y \in P_2$ and $x \in P_1 \setminus P_2$. We get $P_1 \setminus P_2$ and P_2 are disjoint $(1,2)^*pD$-sets containing x and y respectively.

Sub Case (b). $y \notin P_1$ and $x \in P_1 \setminus P_2$ and $x \notin P_3$ implies that $x \in P_1 \setminus (P_2 \cup P_3)$ and $y \in P_3 \setminus P_4$ and $y \notin P_1$ implies that $y \in P_3 \setminus (P_1 \cup P_4)$ and $P_1 \setminus (P_2 \cup P_3)$ and $P_3 \setminus (P_1 \cup P_4)$ are disjoint. Therefore, X is $(1,2)^*$pre-D_2.

Sufficiency. Follows from Remark 4.4. □

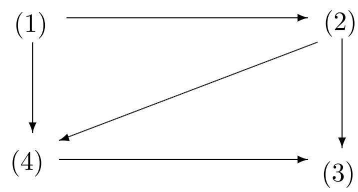
Theorem 4.9. *If X is $(1,2)^*$pre-D_1 then it is $(1,2)^*$pre-T_0.*

Proof. Follows from (ii) Remark 4.4 and theorem 4.8. □

Remark 4.10. An $(1,2)^*$pre-T_0-space is not $(1,2)^*$pre-D_1, in general. In Example 3.10, X is $(1,2)^*$pre-T_0 but not $(1,2)^*$pre-D_1.

Remark 4.11. From the discussions in Sections three and four, the following implication diagram is drawn. In the diagram,

1. $(1,2)^*$pre-T_2
2. $(1,2)^*$pre-T_1
3. $(1,2)^*$pre-T_0
4. $(1,2)^*$pre-D_1



$A \rightarrow B$ (resp. $A \nrightarrow B$) represents that A implies B (resp. A does not imply B).

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